



## Review

# Uncertainty models for stochastic optimization in renewable energy applications



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## ABSTRACT

With the rapid surge of renewable energy integrations into the electrical grid, the main questions remain; how do we manage and operate optimally these surges of fluctuating resources? However, vast optimization approaches in renewable energy applications have been widely used hitherto to aid decision-makings in mitigating the limitations of computations. This paper comprehensively reviews the generic steps of stochastic optimizations in renewable energy applications, from the modelling of the uncertainties and sampling of relevant information, respectively. Furthermore, the benefits and drawbacks of the stochastic optimization methods are highlighted. Moreover, notable optimization methods pertaining to the steps of stochastic optimizations are highlighted. The aim of the paper is to introduce the recent advancements and notable stochastic methods and trending of the methods going into the future of renewable energy applications. Relevant future research areas are identified to support the transition of stochastic optimizations from the traditional deterministic approaches. We concluded based on the surveyed literatures that the stochastic optimization methods almost always outperform the deterministic optimization methods in terms of social, technical, and economic aspects of renewable energy systems. Thus, this review will catalyse the effort in advancing the research of stochastic optimization methods within the scopes of renewable energy applications.

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## 1. Introduction

Renewable energy sectors have seen tremendous growth in the last decade throughout the world especially in Northern America, Western Europe, and China accounting for almost half of the expansion [1]. The recent rapid energy shift in these parts of the world are mainly due to the reduction of production costs of the renewable energy generators, the drive to reduce carbon emissions, and attractive tariffs offered [2]. Wind energy and solar energy accounts for the most rapid growth in renewable energy generations with an approximate 77% of new capacity, with hydropower dominating the rest [3]. Despite being a clean and abundantly available energy (in some parts of the world), renewable energy resources still suffer from its lack of energy density and its intermittency [4]. The latter presents the most challenge to researchers in terms of successfully predicting and utilizing the usage and control of renewable energy resources. In contrast to the conventional generators (i.e. coal or steam turbine power plants), renewable energy generators can only generate energy, when there are renewable resources available. Therefore, appropriate prediction, control, and precise representation of renewable energy systems play an important role to ensure stable and uninterrupted energy supply. Optimization of renewable energy systems can be accurately solved if uncertainties, probabilities, and fluctuating behaviours of renewable energy systems are being properly represented [5].

The current wave of optimization approach in renewable energy applications are shifting. The first wave was in the form of deterministic approaches [6]. During this wave, mixed integer programming has stood out from the earlier modelling approaches namely; dynamic programming, priority list, Lagrangian Relaxation, etc. However, deterministic methods with an assumption of perfect information produced idealistic results which contradicted with the core value of renewable energy systems. With the fluctuations of renewable resources, varying demands, and intermittent economic parameters, deterministic approaches alone could not fully capture the dynamics of the whole renewable energy systems [7].

Studies are now moving towards stochastic optimization in which the optimization considers uncertainties and probabilities as inputs, then evaluate its influence on the output of the system [8]. A stochastic optimization then utilizes these scenario uncertainties in its objective function's formulations. Hitherto, vast amount of literatures has been found regarding the stochastic optimization techniques [9]. Stochastic optimization provides a range of possible solutions which models closer to real – world situations that would benefit operators/consumers in assessing the risks involved in the uncertainties of renewable energy generations. Therefore, the characteristics of stochastic optimization methods are more suitable in handling renewable energy system's fluctuating and intermittent nature. Stochastic optimization however, generally suffers

from huge computational expenses due to large number of scenarios that needs to be considered in its calculations [10]. Numerous techniques have been developed by many authors in increasing the stochastic optimizations' efficacy to reduce computational expenses [11]. Nonetheless, it is still computationally demanding and suffers from the 'curse of dimensionality' in cases of assessing the problems over multivariate and multiple periods of time intervals. Despite the advantages of the stochastic optimizations, its implementations in renewable energy applications are still relatively new. The problems in its transparencies, computational efficacies, and their full practical implementations are still being addressed by system operators and other interested parties.

Based on vast relevant surveys conducted, the paper's motivation is to analyse recent and notable stochastic optimization methods in the lights of renewable energy applications while identifying its current and future research directions. We also pointed out various advantages and disadvantages of highlighted stochastic optimization methods. Given the vastness of stochastic optimization methods that exist hitherto, the focus of the paper is to provide a basic introduction to the highlighted methods while directing interested readers towards notable works of other authors mainly in the field of renewable energy applications.

## 2. Overview of stochastic optimization

The general stochastic optimization in renewable energy applications is broken down into several steps as summarized in Fig. 1. The paper highlights these steps and focuses on the notable stochastic methods in recent renewable energy applications. As stated, the idea of the paper is to provide new researchers as well as advanced readers in the optimization field with insights on the recent and notable stochastic optimization methods in renewable energy applications. The paper is focused on the intuitive part of the stochastic optimization methods rather than the mathematical discourses of the field. Readers are also exposed to the recent trending of the stochastic optimizations in renewable energy applications as well as future works and relevant research themes in these areas. From these recent works, gaps and future works from the literatures are analysed. The trending from the surveyed recent literatures is highlighted and the efficacy of the stochastic optimization approaches is presented from the main results of the literatures.

We consider only the most recent literatures in stochastic optimization methods in the field of renewable energy applications. Key aspects pertaining to the stochastic optimizations are featured such as its main results and contributions, its research gaps, and its uncertainty parameters. The notable mentioned methods are chosen based on its contributions in the field as well as its future implementation prospects. Past authoritative works are also highlighted for interested readers to research further. In this paper, within the scope of renewable energy applications,

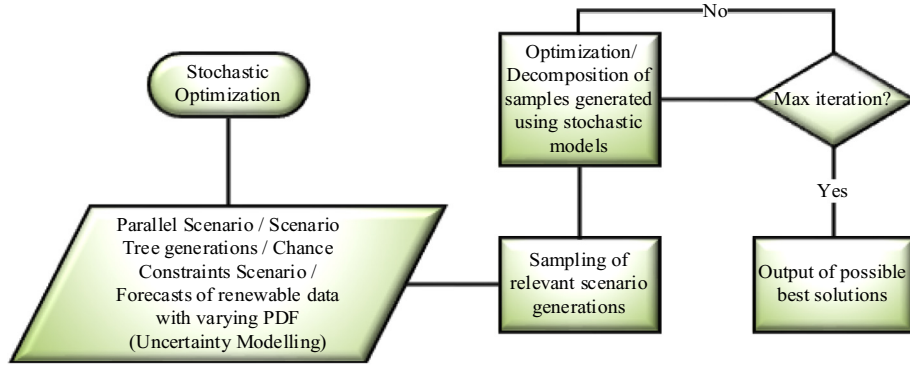


Fig. 1. Stochastic optimization flowchart.

uncertainty modelling/scenario generation approaches are initially addressed, followed by notable sampling methods to capture relevant scenarios in stochastic optimizations. Next, the stochastic optimization approach is highlighted mainly in the lights of approximate stochastic dynamic programming. The paper concludes with main issues and challenges of stochastic optimization approaches in renewable energy applications, followed by its critical remarks and future relevant themes in renewable energy applications.

### 3. Uncertainty modelling in stochastic optimization approach

In stochastic optimizations, representing the correct uncertainties are critical. Each uncertainty modelling technique would yield a different representation of the systems. Therefore, appropriate selection of uncertainty modelling methods is crucial. Uncertainty modelling is a typical way to represent the stochasticity of renewables' systems. Instead of assuming perfect knowledge of the parameters (i.e. wind speed, solar irradiation and load demand) as opposed to a deterministic approach, random distributions are added as inputs to a stochastic optimization approach to mimic the probabilistic characteristics of a renewable energy system. In representing the uncertainties, it is critical that the distribution dynamics of the scenarios are well captured. One of the ways to do that is by generating large number of scenarios, where each scenario would capture the possible realization of the underlying uncertainties. The idea is to find the close approximation of the uncertainties' true distributions. In other words, the main goal is to infer a probability distribution(s) of an output(s) based on a given probability distribution function(s) (PDF) of an assumed known input(s). PDF distributions of inputs varies from parameters

or variables involved. Fig. 2 shows an overview of the uncertainties' modelling approaches. The scope of the paper is only limited to the numerical method of the uncertainty modelling approaches, mainly in the recent Monte Carlo Simulation (MCS) approach in renewable energy applications. Interested readers are encouraged to read the works made by Refs. [11–13] for further information regarding other uncertainty modelling approaches.

#### 3.1. Monte Carlo Simulation

MCS is one of the most used methods in the probabilistic uncertainty modelling approach [15,16]. Historical probability distribution function, forecasting errors, and market variability are the parameters that can be utilized by the MCS method to learn and populate the scenarios' generations. The MCS method is favoured due to its ability to systematically sample from random processes [16]. Furthermore, a transfer function is not necessarily needed in MCS. The problem can be treated as a black box system which can yield related output with given samples of inputs. MCS is also intuitive and relatively easy to implement. MCS can also be implemented in non – differentiable as well as non – convex problems. Apart from that, it supports all probabilistic distribution function (PDF) types. Regardless, MCS has some deficiencies issue such as expensive computation due to its iterative behaviour, especially when the degrees of freedom and the space search expands [17]. The general MCS method in renewable energy applications can be described in Fig. 3.

##### 3.1.1. Types of Monte Carlo Simulation

According to Ref. [18], MCS method is typically divided in three different types. The first one is called the Sequential – MCS method.

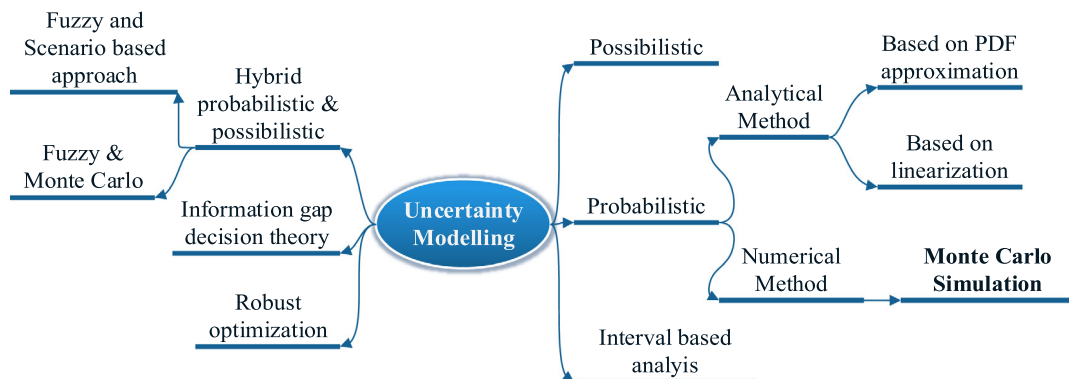


Fig. 2. Uncertainty modelling overview.

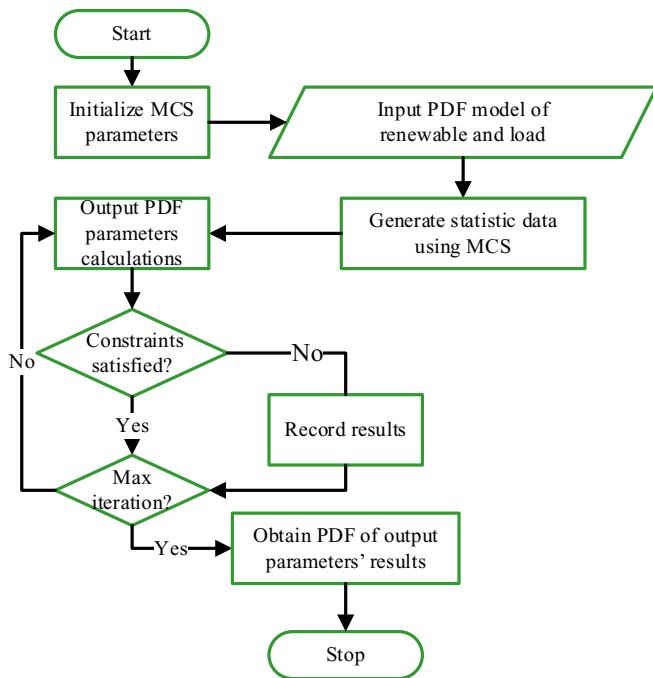


Fig. 3. MCS Flowchart in renewable energy applications.

This method represents the uncertainties in chronological order and is valued for its flexibility for assessing the reliability of the system's posterior distribution. This method is suitable in the applications of time series of variable energy sources and variable load. Nonetheless, it requires a significant amount of computational effort as the dimensions of the uncertainties grow due to sequential and iterative characteristics of this MCS. This method also may be infeasible for some applications which are non-sequential. The second type of the MCS is the Pseudo-Sequential MCS. It is named due to its ability of non-sequential sampling of system states and chronological simulation of only the sub-sequences tied with the failed states. This method has a faster convergence rate than the sequential-MCS method. However, this method is still demanding in terms of computational expenses as the degree of the problem increases. The third and the last type is called the Non-Sequential MCS method. It is known for its high computational efficiency but lacks the ability to simulate the chronological aspects of a renewable system operation. The summary of the three types of the MCS method mentioned with respect to the literatures mentioned is shown in Tables 1–3.

### 3.1.2. Recent MCS applications in renewable energy

**3.1.2.1. Sequential MCS applications in renewable energy.** The sequential MCS method has been implemented as follows in renewable energy applications [19]. implemented the MCS in scenario generations of irradiation, wind speed, load, and temperature as inputs to optimize for the control of PV-wind-diesel-battery stand-alone systems. Authors suggested that a reduction in search space is recommended to optimize the problem within acceptable time frames. Genetic Algorithm (GA) has been used initially to reduce the possible scenario generations' space of the MCS. A novel hybrid GA and MCS approach was proposed by Ref. [20] to predict hourly energy consumption and generation by a cluster of Net Zero Energy Buildings. The MCS aspect considers the variability and the modelling aspects of the random energy consumption in a building at a given specific hour. An analytical convolution process combined with MCS in the works of [21] to

determine the optimal amount of power generation to be committed by incorporating renewable power forecasting errors and system reliability. A study by Ref. [22] proposed a sliding time window optimization approach to find an optimal design and dispatch scheduling strategy in a hybrid renewable energy system consisting of biomass, wind, solar, gas-fired boiler, battery, and thermal energy storage. Different scenarios were created to find out which combinations of renewable generators would yield the optimum design and dispatch strategy. MCS was then tasked to generate the cost of energy (COE) distribution as an output to provide a risk indication of the chosen design. MCS has been utilized by Ref. [23] to sample different system states from the self-adapted evolutionary strategy (SAES) combined with Fischer-Burmeister algorithm in optimizing the one-time investment and the operational costs of hybrid wind-energy storage power system. Lopes et al. (2015) have addressed the impact and system reliability on the combination of wind generation and small hydro power plants [24].

The uncertainties of power generations are modelled using the MCS. A novel risk management method was investigated in the work of [25] based on managed charging of plug-in hybrid electric vehicle (PHEV) and vehicle-to-grid (V2G) using MCS. MCS analysis of wind farm lightning surge transients aided by a lightning detection network data is implemented by Ref. [26] to produce a statistical depiction of over-voltages distribution within the wind farm electrical network. The statistic depiction can be used to assist wind farm lightning risk management and surge protection optimization. MCS has been implemented by Ref. [27] to consider the uncertainties of load and irradiation in the economic optimization of energy supply at off-grid healthcare facilities. In the work of [28], the authors have utilized MCS by performing Temperature-Augmented Probabilistic Load Flow (TPLF) to characterize the aspects of over-limit probabilities of events such as over and under-loading of loads and voltages in a 39-bus test system.

#### 3.1.2.2. Pseudo-sequential MCS applications in renewable energy.

The MCS method handled the uncertainties which are; the solar irradiance which is modelled using kernel density estimation, the load demand using a Gaussian distribution, the wind speed using a Weibull distribution. In the work of [29], the author coupled MCS with quantile estimation techniques, and an efficient stochastic optimizer, Adaptive Global Local Search (AGLS) in sizing hybrid renewable energy systems while considering the renewables uncertainty as inputs to MCS. Authors found that the approach has enabled the control of the upside risk, consequently enhancing the decision quality regarding the hybrid renewable energy systems. Implementation of MCS in Ref. [30] has been represented to show the possible distribution of thermal energy collected at a solar thermal power plant. Applying a pseudo-MCS reduce the search space of the non-convex stochastic optimizer which is the PSO, to find an optimal design that leads to an improvement of yearly thermal energy collected between 3.34% and 23.5%. MCS has been applied in Ref. [31] to consider the intrinsic variability of electric power consumption in the probabilistic assessment simulation of DG penetration in medium voltage distribution networks. The fluctuations and uncertainties of load demands and generations of solar PVs are represented using MCS in the work of [32]. A multi-linear MCS method is proposed by Ref. [17] to analyse the steady state operating conditions of an active electrical distribution system with Wind and PV generation plants. The uncertainties of power load demand and power production from renewable generations are considered using the MCS combined with multi-linearized power flow equations. In the work of [33] MCS is implemented to model the uncertainties of energy demand, solar energy availability, and electricity prices followed by a space search

**Table 1**  
Sequential MCS in renewable energy applications.

References	Method	Objective function	Type of MCS	Uncertain input Parameters	Main results/contributions	Future work/Gaps
[19]	MCS – GA	Minimize investment and operational costs	Sequential	Solar irradiation, Temperature, Wind speed, Annual fuel price interest rate, Average daily load	More information available on expected performance and costs of the system with respect to the deterministic optimization	Optimize MCS samples and computational time
[20]	GA – MCS	Minimize instantaneous and cumulative energy balance	Sequential	Buildings' energy consumptions	Reduction in net energy balance in buildings	Extending the method's period to more than one year
[21]	MCS – Analytical Convolutional Process	Optimize cost/benefit relationship of RE generations	Sequential	Wind speed forecast error, generation unit reliability	Considerable improvement of computational efficiency with reasonable cost/benefit	Increment of MCS computational efficiency
[22]	Receding Horizon Optimization – MCS	Minimize Cost of Energy, minimize risks	Sequential	Wind power, Solar power, Battery storage, Biomass combined Heat Power, thermal energy storage, gas producer	Lowest cost option may have a higher risk of failing. The model provides ranges of possible microgrid designs to determine major risk factors	Comparison of short-term performance with while considering demand side uncertainties
[23]	SAES – ARM –MCS	Minimize investment and operational costs	Sequential	Load demand, Wind speed	Reduces iteration in a complex search space; Investigate discharge cycle efficiency of different energy storage on the system	Investigate impact of energy management on planning decisions
[24]	Risks based – MCS	Minimize loss load probability (LOLP), EPNS, LOLD and LOLF	Sequential	Wind speed, River inflows	Precise estimation of energy delivered at a given time and reducing load shedding risks	Integration of various intermittent RESs
[25]	LOEE – MCS	Minimize loss of energy expected and expected energy not supplied	Sequential	PHEV owner's behaviour, Solar and Wind power	LOEE in novel charging applications reduced by 75% in comparison to unmanaged charging	N/A
[26]	LINET - MCS	Mitigation of lightning risks	Sequential	Lightning transients/activity at wind turbines	Cost – effective overvoltage protection selection	N/A
[27]	NPC – MCS	Minimize net present cost (NPC)	Sequential	Load, Solar irradiation	Realistic stochastic battery lifetime prediction using weighted Ah Schiffer method	Analysing cost reduction and fossil fuel consumption, Improving supply reliability
[28]	TPLF - MCS	Minimize risk of system over – voltage and risk of system over – load	Sequential	Load, Solar irradiation, Solar PV output, Temperature	Accurate uncertainty modelling of Solar PV output, load, and temperature at multi – time instants	Considering multi – time spatial and temporal correlations in power generation dispatch strategy

**Table 2**  
Pseudo - Sequential MCS in renewable energy applications.

References	Method	Objective function	Type of MCS	Uncertain input Parameters	Main results/contributions	Future work/Gaps
[29]	MCS – AGLS	Minimize risk	Pseudo - Sequential	Possible sizing of HRES	Sizing of HRES with minimal risk	An efficient quantile estimation method to solve large-scale problems
[30]	Ray Tracing MCS – PSO	Maximize yearly thermal energy collection	Pseudo – Sequential	Sun ray's position, days of the year	Increment in yearly thermal energy collected	Integration of electrical output in the system, optimization of levelized cost of energy
[33]	Multi objective – Roulette Wheel – MCS	Minimize energy costs and environmental impacts	Pseudo – Sequential	Supply side; Demand Side, domestic hot water, space heating and cooling)	Models and proposed methods provided accurate optimization results in identifying the economic/ environmental pareto fronts	N/A
[36]	Cholesky Decomposition – MCS	Minimize economic risks and maximize financial returns	Pseudo – Sequential	water inflow, wind speed, solar irradiance, temperature of PV panels, and average generation capacity	Main characteristics of the random variables are accurately modelled for energy applications	Test proposed method with plant's installation site data (real data); Adaptation of the method in other markets
[34]	Various techniques – MCS	Minimize LOLP, Expected Unserved Energy (EUE)	Pseudo - Sequential	Load demands, Conventional generation resources, Wind resources	Quantify the impacts of integrated renewable resources on reliability, power economics, and emissions	Integration of other stochastic parameters such as solar, demands, and storages



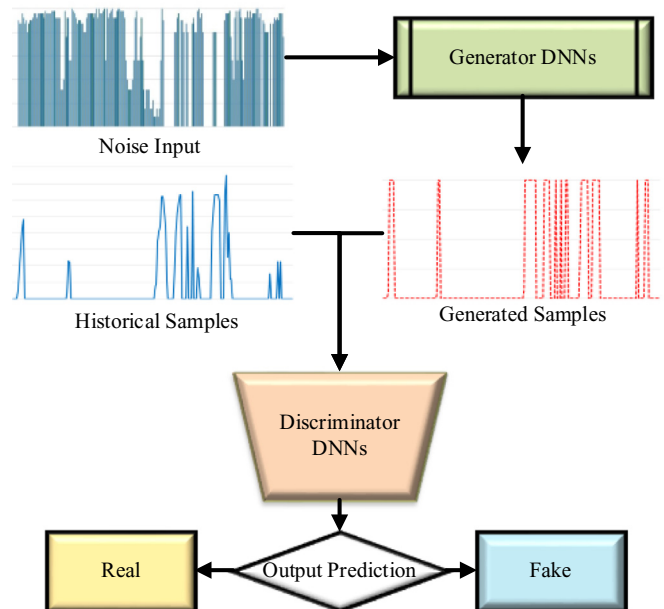
**Table 3**  
Non - Sequential MCS in renewable energy applications.

References	Method	Objective function	Type of MCS	Uncertain input Parameters	Main results/contributions	Future work/Gaps
[35]	NPV - MCS	Optimize NPV	Non - Sequential	Risks and finances parameters (refer to the paper for further clarifications)	Optimization of conceptual design with respect to investments, security, and returns	Generalization of analysis method where correlations between parameters are permissible
[37]	LCA - MCS	Evaluate random variables environmental impact	Non - Sequential	Well fluid composition, Drilling time, geothermal well life	Main environmental impacts of geothermal plants are shown	Comparison of enhanced geothermal plants LCA with traditional plants
[38]	Random MCS (RMCS) with annual branch fault & Sequential MCS	Minimize system interruption; SAIFI, CAIDI and SAIDI (refer to paper)	Non/Sequential	Outage per year, duration of outage	Proposed method outperforms sequential MCS in terms of risks analysis	N/A

reduction technique (Roulette – Wheel Mechanism) to reduce the computational expenses. Pinheiro et al. (2017) has implemented MCS associated with Cholesky Decomposition as inputs to generate synthetic time series of water inflow, wind speed, solar irradiance, temperature of PV panels, and average generation capacity [34]. The MCS's goal is to perform risk analysis with 2000 scenarios that spans over a period of 300 months. Authors in Ref. [34] deployed the MCS to systematically sample random processes of intermittent renewable resources and simulated the power system and transmission constrained day – ahead market operations. MCS approach to investigate the economic risk analysis of decentralized renewable energy infrastructures has been used in the work of [35]. The MCS method considers the net present value (NPV) estimation and its ranges for each scenario involved.

**3.1.2.3. Non – sequential MCS applications in renewable energy.** Few recent works that have implemented the non – sequential MCS in renewable energy applications are mentioned as follows. The MCS approach to investigate the economic risk analysis of decentralized renewable energy infrastructures has been implemented in the work of [35]. The MCS method considers the NPV estimation and its ranges for each scenario involved. Hanbury and Vasquez, 2018 employed the usage of MCS in geothermal plant's construction to stochastically capture the environmental impact in terms of complete Life Cycle Analysis (LCA) relative to other methods of energy production [37]. A systematic approach of MCS to address the system distribution reliability considering intentional islanding was implemented in the work of [38].

**3.1.2.4. Trending of MCS applications in renewable energy.** Trending of MCS applications in renewable energy applications is hybridized with either a (meta) – heuristic method, strategic sampling methods, or other optimization methods. The (meta) – heuristic method typically acted as a space search reducer for the MCS method in performing the stochastic optimization as shown in Ref. [19], thus decreasing the overall computational expenses. Other method such as sampling methods (Typically related to



**Fig. 4.** Example of GANs structure for wind scenario generation.

Pseudo – Sequential MCS) has strengthened the weakness of MCS method that requires large amount of sampling data to be accurate [38,39]. With strategic sampling of the scenario generations only the most important components are considered that contributes the most to the objective function's stochastic optimization. Due to the time dependent nature of renewable energy problems, most of the applications of MCS methods in this field is either Sequential or Pseudo – Sequential. **Section 4** describes further methods involved in stochastic optimization sampling processes.

### 3.2. Notable uncertainty modelling method: Generative Adversarial Networks

It is important to note that as computational advancements have been growing rapidly throughout the years, model – driven uncertainty modelling or scenario generations' methods have been becoming less viable, difficult to apply, and hard to scale [15,41]. These are caused by the complex dynamics of renewable energy systems, time – varying nature of weather, and complex temporal and spatial connections. Studies are now converging towards the data – driven methods in generating new sets of unique and distinctive scenarios in renewable energy application [41]. As one

day might not be the same as another due to erratic weather changes and global warming, a new method can't only rely on generating/projecting scenarios based on historical data but must also correctly capture the rapid variations and strong diurnal cycles of renewable resources in generating authentic new scenarios. Numerous amounts of literatures exist in scenario generations of renewable resources such as wind and solar as well as demands [42]. However, most of them were model – driven and it is cumbersome to pin point the most efficient usage of an exact model to an exact situation. A recent study derived from Artificial Neural Networks (ANNs), namely Generative Adversarial Networks (GANs) by Ref. [43] has been gaining a lot of attention due to its ability to synthesize artificial images from trainings of real ones. Only few works have been identified in literatures that implemented data – driven GANs in renewable energy applications. The method successfully synthesizes renewable system's scenarios in Ref. [41] using Wasserstein GANs. The generated scenarios are successful in synthesizing new and distinct scenarios by capturing the intrinsic features of the historical data.

Fig. 4 depicts an overview of GANs system. The intuition behind GANs is to exploit the capacity of Deep Neural Networks (DNNs) in both classifying complex signals (Discriminator) and expressing

**Table 4**  
Characteristic and benefits of different types of GANs.

References	Type of GANs	Characteristic	Main advantage(s)	Future work(s)
[45]	Wasserstein GANs	Using the Earth – Mover distance to evaluate the distribution gap between real and generated data	<ul style="list-style-type: none"> <li>Stable training of GANs</li> <li>improves the learning parameter and optimization method of conventional GANs</li> </ul>	<ul style="list-style-type: none"> <li>Developing new algorithms for calculating Wasserstein distance between different distributions</li> </ul>
[46]	Loss – Sensitive GANs	Limiting the modelling ability of the discriminator to better distinguish the real and generated samples regardless of their complexity	<ul style="list-style-type: none"> <li>Reduces over fitting of generated samples</li> <li>improves the learning parameter and optimization method of conventional GANs</li> </ul>	<ul style="list-style-type: none"> <li>N/A</li> </ul>
[47]	Semi - GANs	Adding labels of real data to the training of discriminator	<ul style="list-style-type: none"> <li>Generates a higher quality sample than conventional GANs</li> <li>Reduces training times for the generator</li> </ul>	<ul style="list-style-type: none"> <li>Weighting of discrimination and classification</li> <li>Generating examples with class labels</li> </ul>
[48]	Bidirectional GANs	Mapping the real data to the latent variable space in an unsupervised learning environment	<ul style="list-style-type: none"> <li>No assumptions of underlying structure of data are needed</li> <li>Outperforms many unsupervised feature learning approaches</li> </ul>	<ul style="list-style-type: none"> <li>Testing of the Bidirectional GANs under other space of architecture models</li> </ul>
[49]	Info GANs	Capturing mutual information between a small subset of latent variables and observations	<ul style="list-style-type: none"> <li>Learns interpretable and disjointed representations on challenging datasets completely unsupervised</li> <li>Negligible increment in computational expenses compared to conventional GANs and easy to train</li> </ul>	<ul style="list-style-type: none"> <li>Applying mutual information and induce representation to other methods such as variational autoencoder</li> </ul>
[50]	Auxiliary – Classifier GANs	Incorporating label information into the generator and adjusting objective function for the discriminator	<ul style="list-style-type: none"> <li>Generation and discrimination capability of GANs are enhanced</li> <li>Produces a more diversified samples of data</li> </ul>	<ul style="list-style-type: none"> <li>Improving the reliability of the proposed GANs</li> <li>Improving visual discriminability</li> </ul>
[51]	Sequence GANs	Generating sequences of discrete tokens	<ul style="list-style-type: none"> <li>Excellent performance in synthesizing speech, poem, and music generation</li> </ul>	<ul style="list-style-type: none"> <li>Monte Carlo tree search in improving the action decision making for large scale data in cases of long – term planning</li> </ul>
[52]	Boundary – Equilibrium GANs	Equilibrium enforcing method paired with a loss derived from the Wasserstein distance	<ul style="list-style-type: none"> <li>Balances the discriminator and generator in training</li> <li>Provides trade – offs between samples' diversity and quality</li> </ul>	<ul style="list-style-type: none"> <li>Determining the best latent space size for a given dataset</li> <li>Determining when and how noises should be added to the input</li> </ul>

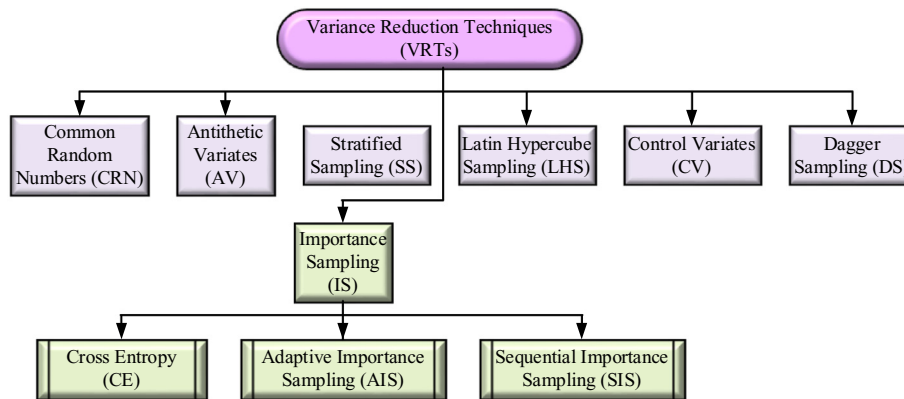


Fig. 5. Overview of variance reduction techniques (VRTs).

complex non – linear interactions (Generator). The idea behind GANs is to set up a minimax game of two DNNs which are in an adversarial relationship with each other. The Generator's DNNs updates its weights during each training epochs to “trick” the discriminator by generating “fake” samples of scenarios, while the Discriminator's DNNs attempts to distinguish between true historical scenarios and the “fake” ones. Theoretically, after reaching equilibrium, the optimal solution of GANs will yield scenario distributions from Generators which are hardly distinguishable from an authentic real historical data.

Hence, the Discriminator can no longer differentiate the origin of the data, whether it is from the generator or the real historical training data distributions. It is easier to imagine the GANs as a counterfeiter and a police game where the counterfeiter (Generator) keeps on improving its technique to deceive the police, while the police (Discriminator) are also getting better at catching the counterfeiter. The counterfeiter in the end would produce a “fake product” that resembles the authentic product successfully, which no longer can be identified by the police.

To summarize, the GANs method in scenario generation can leverage the power of DNNs and vast sets of historical data in performing the tasks for directly generating scenarios conforming to the same distribution of historical data, without the need of explicit modelling of the distribution [43]. However, it is important to note that the architecture of DNNs is complex in nature and requires high computational efficacy in solving GANs problems. Yize et al. in his work [41] has suggested the usage of efficient GPU(s) to accelerate the DNNs training procedures. Future works in renewable energy systems using GANs would be in the decision – making strategy for unit commitments with high penetration of renewable energy generations and incorporation of the method in probabilistic forecasting problems. Interested readers are directed to the work of [44] for a comprehensive overview of GANs as well as its future trending. Hitherto, several main variants of GANs have been identified and are summarized in Table 4. Characteristics, main advantages, and identified future works of the GANs' variants are highlighted for the perusal of interested readers. It is to be noted that the future works identified are mostly in the realms of computational and mathematical sciences. However, implementations of these GANs' variants in renewable energy systems are yet to be tested.

#### 4. Sampling methods in scenario generations

Increasing scenario generations would intuitively mean a closer and more comprehensive representation of possible futures. Nonetheless, increment of scenario generations (samples taken)

might only marginally increase the quality of the solution and the objective function until a certain threshold [6]. One need to carefully evaluate the trade – offs between the accuracy and the rate of convergence of a given algorithm. One popular technique to increase the sampling precision is called Variance Reduction Techniques (VRTs) [12]. VRTs can be broken down into several main sub – categories as shown in Fig. 5. The estimates of scenario generations' precision depend on standard deviation between the samples. The standard deviation can be expressed in equation (1) below:

$$\sigma = \frac{\sqrt{V(z)}}{\sqrt{N}} \quad (1)$$

where  $V(z)$  the unbiased sample variance and  $N$  is the sample number.

According to equation (1), the precision of the estimates can be intuitively increased by increasing the number of samples,  $N$ . However, increasing the samples' size would mean reducing the efficacy of computation. In cases of sequential sampling process throughout a year, with 8760 h steps, each hour containing its own multivariate properties, a sample increase of 1 would mean a repetition of 8760 h of sampling process. Therefore, another way to keep the sample size small yet still maintaining a desired precision is to reduce the variance between the samples. The main idea behind VRTs is to decrease the amount of sampling needed to the desired level of accuracy or increasing the accuracy of the expected value for a given number of samples. There are various VRTs which have been reported in literature in renewable energy applications, as depicted in Fig. 5.

The authors in Ref. [53] used a range of random variables (RV) to develop an improved stochastic model for power system scheduling in the presence of uncertain renewables. A work in Ref. [54] focused on reliability evaluation through sequential Monte Carlo simulation to address cascading failure in power systems operation. The Weibull distribution together with antithetic variates (AV) is implemented in order to reduce the large computational burden in simulations. Karadooni et al. [55] conducted a survey on climate change and renewable energy in Peninsular Malaysia based on stratified sampling (SS). The authors in Ref. [56] identified the factors shaping public opinion based on random stratified sampling to examine willingness to pay for expansion of renewable energy sources in the electricity mix. A novel modified Latin hypercube-important (LHS) sampling method is suggested in Ref. [57] to enhance the accuracy and speed of correlation processing under low sampling times. A LHS method is proposed in Ref. [58] to analyse the reliability of power systems considering the



intermittent behaviour of renewable generations such as wind, solar power and fluctuation of bus loads. Dahlblom [59] applied control variates (CV) for Monte Carlo-pricing on three-asset spread options with a view towards energy markets. A control variable based dagger sampling (DS) technique is proposed in Ref. [60] to decrease the computational effort in Monte-Carlo reliability evaluation for composite systems. Apart from the methods mentioned above, Importance Sampling (IS) Method has become popular in renewable energy applications. IS method and sub-division are explained in the following section.

4.1. Importance Sampling (IS) method

Recent surveys from the literatures have shown that the IS method boost the sampling efficiency [61]. Typically, in MCS, the sampling representations would be excellent, if and only if samples can be drawn from the target distributions. However, certain rare cases in renewable energy applications such as extreme wind cycles, sudden power outages, and rare occurrences of device failures are difficult to account for. IS method focuses on sampling the important region (usually named “proposal distribution”) in which the important region have greater occurrence probabilities in comparison to the original distribution. The intuition is to construct a proposal distribution that “boosts” the sampling of important regions. The method can bring enormous advantage, making an otherwise seemingly impossible problem for typical MCS, amenable. Nonetheless, applying IS method requires experience in sampling due to its double – edged characteristics. One could easily go wrong by yielding an estimate with infinite variance, when a simple sampling method could have yielded a finite one. Therefore, a well – chosen proposal distribution is the key to maximize computation efficiency.

4.1.1. Type of IS method

“Trainings” are encouraged with a trial distribution to capture the appropriate estimate distributions using MCS. With repetitions of MCS simulations, a better trial distribution can be drawn out based on the weighted MCS samples. The process is repeated until termination criteria are met. The “Trainings” and the trial distribution procedure is called the Adaptive Importance Sampling (AIS) method, as the proposal distribution is updated adaptively. Another typical form of IS is called the Sequential Importance Sampling (SIS). As the name suggests, SIS constructs the proposal distribution sequentially and typically requires a decomposition procedure. SIS is normally implemented in high – dimensional problems in building up proposal distributions sequentially. Cross – Entropy (CE) was proposed by Ref. [62] to enable the inclusion of very unlikely events in computations. CE is a popular sub – category of IS method in VRTs to account for the optimizations of rare events [63]. Based on repeated sampling, the method utilizes each iteration in two steps; random data generation using a specific random method and updating the specific method’s parameters to yield an improved sample in the next iteration. According to Ref. [64], IS method is the hardest variance reduction method to use, therefore expertise in the field is a necessity. Readers are advised to read the works of [46] for the detailed mathematical representations and implementations of IS methods. The following paragraph briefly presents the recent literatures in IS, CE and SIS applications in renewable energy systems. From our extensive literature searches, only a few recent literatures existed in the implementation of IS in renewable energy applications.

4.1.2. IS method implementations in renewable energy

IS in reducing the computational time of MCS has been implemented by Ref. [65] in a probabilistic security management for power system operations with large amounts of wind power.

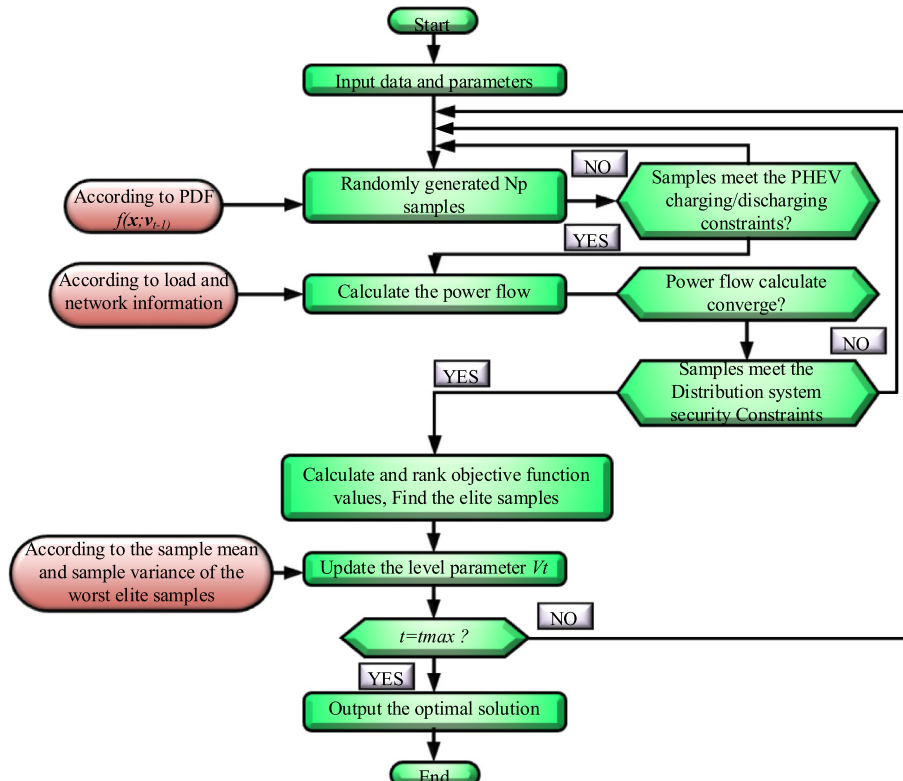


Fig. 6. Framework of CE based dispatch model to handle uncertainties in PHEVs and renewable generation [66].

Author has found out that the IS method significantly reduced the computational time needed in sampling by two to three orders and has shifted the original distribution to the desired proposal distribution. In the work of [66], CE method is utilized in hybrid renewable generation's optimal dispatch strategy of Plug – in Hybrid Electric Vehicles (PHEVs) to improve the voltage profile and the power flow with a 33 – nodes distribution systems. Authors have found that the proposed method has managed to decrease the power flows in heavy loaded lines and renewable generation fluctuations. The objective function is developed using two parts. The first part presents the expectation of population variance of renewable generation outputs, while the second part denotes the expected operation cost including battery degradation, PHEV owner benefits and control of the fleet of the vehicles. The objective function is formulated as follows:

$$Min \alpha_1 E [D_p(P_{e,t})] + \alpha_2 E \left[ C_{BD_p} \sum_{t=1}^T (1 + u\%) P_{dch,t} \Delta t \right] \quad (2)$$

Where,  $E$  denotes the outputs of PV system/wind generator,  $C_{BD_p}$  is the per MWh cost of the battery degradation,  $P_{dch,t}$  is the average power consumption of a single PHEV in time interval  $t$ ,  $P_{e,t}$  is the total charging/discharging power of PHEVs in time interval  $t$ ,  $\Delta t$  is the time interval,  $\alpha_1$  and  $\alpha_2$  are the probability density function (PDF) parameters.

The comprehensive framework is developed with the multiple cases such as typical situations of seasonal renewable generation and vehicle usage, as shown in Fig. 6. Different renewable market share and peak generation or demand circumstances are also discussed.

An efficient sampling method for MCS in Ref. [67] has been investigated using CE and Copula theory to analyse generation adequacy of multi – area power systems with high penetrations of wind power. Results have shown that the sampling method significantly reduced the number of samples required to estimate reliability parameters of interest. A robust Multi – Objective CE (MOCE) algorithm is proposed by Ref. [68] in integrated scheduling approach to solve for microgrid supply and demand scheduling problem under uncertainties. A multi-objective function is developed using fuel price, maintenance cost, buying and selling electricity price, depreciation cost of battery and penalty cost which can be presented in the following equation,

$$F_1 = \sum_{t=1}^{|T|} \left\{ c^{fuel} f_t^{mt} + \sum_{dg \in A} c^{dg} p_t^{dg} + (z_t^{pg} c_t^{buy} p_t^{pg} + (1 - z_t^{pg}) + c_t^{sell} p_t^{dg}) + s^{bt} p_t^{bt} + (s^{es} |p_t^{es}| + s^{chs} |p_t^{chs}|) \right\} \quad (3)$$

where  $c^{fuel}$  is the natural gas fuel price,  $f_t^{mt}$  is waste heat by burning natural gas,  $A$  is DG unit set,  $c^{dg}$  presents the DG unit maintenance cost,  $p_t^{dg}$  is DG power output,  $c_t^{buy}$  and  $c_t^{sell}$  denote the buying and selling electricity price, respectively,  $p_t^{pg}$  is power grid power output,  $s^{bt}$  is depreciation cost of battery,  $s^{es}$  and  $s^{chs}$  are the penalty cost of shortage/excess electricity and cooling/heating respectively, and  $p_t^{bt}$  is the battery power.

Another objective function  $F_2$  containing coal and natural gas combustion emissions, can be expressed as follows

$$F_2 = \sum_{t=1}^{|T|} (e^{pg} p_t^{pg} + e^{fuel} f_t^{mt}) \quad (4)$$

where  $e^{pg}$  and  $e^{fuel}$  denote the conversion factor of carbon

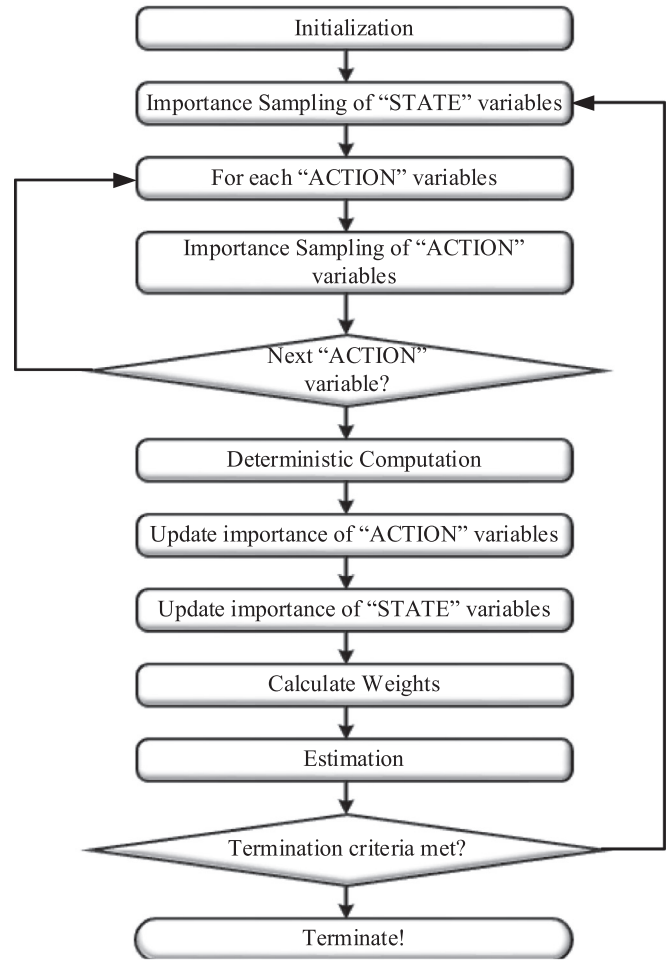


Fig. 7. Flowchart of SIS based hybrid probabilistic method for electricity market [71].

emissions generated from electricity and natural gas, respectively.

Authors have shown that the proposed algorithm has managed to simultaneously minimize operation costs and emissions under the worst – case scenario of fluctuating renewable generations and uncertain loads.

Leite and Castro [69] has presented a new probabilistic method in evaluating spinning reserve margins using CE in renewable energy systems with transmission restrictions. CE is utilized in treating the rare events and identifying necessity equipment for operation in such events. Authors have shown that the CE method was successful in managing higher penetration of renewable sources and ensuring a reliable operation. Graf, et al. [70] has utilized the Adaptive Stratified Importance Sampling (ASIS) method in hybrid extrapolation and MCS method for estimating wind turbine extreme loads. Authors have shown that the variance of the hybrid method are reduced swiftly with the implementation of the ASIS. The minimal variance importance distribution can be derived as follows,

$$q^*(x) = \frac{Y(x)f(x)}{E_f[Y(x)]} \quad (5)$$

$$E_f[Y(x)] \sim \frac{1}{M_{tot} \sum_i Y(x_i)}, \text{ with } x_i \text{ drawn form } f \quad (6)$$

**Table 5**  
IS, CE and SIS implementations in recent renewable energy applications.

References	Method	Objective	IS/CE distribution parameters	Main Results	Future work/Gaps
[70]	Adaptive stratified – IS	To estimate wind turbine extreme loads	Extreme loads, wind speed	The proposed method outperforms sample – based IS – MCS method	Root causes of extreme response variation in wind turbine loads
[65]	Risk assessment – IS	To estimate very low operating risks in power systems	Load, Wind power	Decrease in computational expenses of two to three orders of magnitude with respect to crude MCS	Robustness tests with different values of controllable active power outputs and wind power forecast distributions
[66]	Normal Distribution Parameterized – CE	To provide an optimal dispatch strategy for PHEV	PHEV's driving behaviour, wind speed, solar irradiance, system, and load data	With introduction of Vehicle 2 Grid (V2G), PHEV could act as storage devices and proposed CE models solved for multiple patterns of seasonal profiles for PHEV dispatch cases	Consider future studies intervals in seconds and minutes relevant to power markets like spinning reserves
[67]	Copula Theory & CE	To analyse generation adequacy of multi – area power systems with high penetrations of wind power	Conventional generation, Load, Wind power generation	Proposed method outperforms crude MCS in terms of efficiency and accuracy by three to four orders. Number of samples required does not increase with the decrease of probability interests' level	N/A
[68]	Multi – objective – CE	To schedule energy supply and demand in integrated scheduling under uncertainty	Load profiles, Solar PV power	Total cost and carbon emissions are significantly reduced using proposed method	Large scale integration of distributive resource and renewable energy in regional integrated energy systems
[69]	MCS – CE	To assess probabilistic spinning reserve considering renewable resources and transmission restrictions	Wind generation capacity, Equipment failures, capacity limits of transmission equipment	Using risk assessments and knowing the critical elements of the system, planners can better manage the high penetration of renewable sources, ensuring sustainable and reliable operation	The configuration of the Brazilian interconnected system to demonstrate the practicality of the proposed approach
[63]	MCS-SIS	To assess the deviation of price, possible occurrence of price spike in electricity market	System load, renewable energy output, generator bidding strategy, and outage rate	Estimations for both expected normal price and price spike probability are accurate and fast using less than 3% of the MC simulation time.	The proposed is promising to be implemented on online applications.

Then for any given load  $Y_j$ , The estimation of ASIS can be expressed as follows

$$P(Y < Y_j) = E_f [I(Y < Y_j)] \tag{7}$$

$$= \int I(Y(x) < Y_j) f(x) dx \tag{8}$$

$$= \int I(Y(x) < Y_j) \frac{f(x)}{g(x)} g(x) dx \tag{9}$$

$$= \int I(Y(x) < Y_j) \frac{f(x)}{g(x)} g(x) dx, \sim \frac{1}{M_{tot}} \sum_i I\left(Y(x_i) \frac{f(x_i)}{g(x_i)}\right) \text{ with } x_i \text{ drawn from } g \tag{10}$$

$$= \frac{1}{M_{tot}} \sum_i \begin{cases} \frac{f(x_i)}{g(x_i)} & \text{if } i < f \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

$$= \frac{1}{M_{tot}} \sum_{i < j} \frac{f(x_i)}{g(x_i)} \tag{12}$$

where,  $q^*(x)$  is the auxiliary importance variable,  $Y(x)$  is load,  $f(x)$  is the distribution of wind speed,  $E_f$  denotes expectation with regard to  $f$ ,  $M_{tot}$  is the total number of samples and  $g(x)$  is the arbitrary

distribution.

Huang et al. [71] established a hybrid probabilistic assessment method based on SIS for electricity market risk management. The proposed method has considered various uncertainties such as system load, renewable energy output, generator bidding strategy, and outage rate. The performance is checked under Australian National Electricity Market consisting of 59 buses, with 38 conventional generation units and one wind farm. The authors have found that the method has resonance accuracy similar to MCS results and fast execution with regard to normal price and price spike probability. The implementation flow is illustrated in Fig. 7 where system load is classified into “STATE”, and reported price of each unit into “ACTION”.

Vast amount of recent literatures pertaining to recent IS adaptations and improvements have been found outside of the renewable energy applications which has proven to be efficient and robust to implement, mainly in the fields of signal processing and computational sciences. Recent adaptations of various IS methods in the renewable energy applications are still scarce. Readers are encouraged to read the work of [39] which provides a comprehensive overview of IS methods. In this work, the IS methods' (mainly AIS) scopes are discoursed at great depths from the past, the present, and on to the future. Future works in IS involves the implementations of proposed IS methods with different and wide ranges of distribution parameters in high dimensional problems in which the characteristics of the problems are very similar to the renewable energy applications. IS method's promising new applications involves utilization of the method in the deep learning field for computing the weights of hidden layers.

Summary of the references mentioned above is specified in Table 5.

4.2. Notable sampling method: Markov Chain Monte Carlo method

The Markov Chain Monte Carlo (MCMC) is a popular and rapidly growing sampling method which combines the properties of Markov Chain and MCS [72]. The intuition behind the Markov Chain is to generate random samples through a special sequential process. The next generated random sample depends only on its previous random sample and not affected by any samples prior to the previous ones, thus creating a chain of random generated samples until the end of iterations. This is the well – known “Markov” property. MCMC proved to be advantageous especially in Bayesian inference due to the difficulty of predicting the posterior distributions via analytic methods. MCMC grants the user the ability to approximate the posterior distribution, with minimal number of samples [73].

4.2.1. Overview of MCMC

A simple yet concise introduction to MCMC was written by Ref. [72]. The goal of the authors was to demystify MCMC sampling method and provide a comprehensive example to encourage new researchers/users in adopting the MCMC method for their own research purposes. Interested readers are directed to the work of [74] for in depth analysis and advanced coverage of MCMC. A more technical approach of MCMC method can be found in the work of [75].

In recent renewable energy applications, the MCMC method has been implemented as follows. MCMC simulation model has been utilized by Ref. [76] to consider the uncertainties of renewable energy generation outputs and plug-in electric vehicle (PEVs) charging demand in a combined resource allocation framework in distributed energy storage systems (DESS). The objective function is expressed as follows:

$$\min_{\Omega_1, \Omega_2} \sum_s \text{Pro}(s) \times \left( \sum_i [C_{RE(i)} - R_{RE(i,s)} + C_{ES(i)} + C_{EV(i,s)}] + [C_{loss(s)} + C_{cons(s)}] \right) \tag{13}$$

Where

$$C_{RE(i)} = C_{RE(i)}^{kW} \times P_{RE(i)}^{Cap} / LV \tag{14}$$

$$R_{RE(i,s)} = \sum_d N_{A(d)}^{days} \sum_h \rho_{RE(i,h,d,s)}^{kWh} \times P_{RE(i,h,d,s)} \tag{15}$$

$$C_{ES(i,s)} = \left( C_{ES(i)}^{kW} P_{ES(i)}^{Cap} + C_{ES(i)}^{kW} E_{ES(i)}^{Cap} \right) / LV + C_{ES(i)}^{OM} \tag{16}$$

$$C_{EV(i,s)} = C_{CH} \chi_{EV(i)} / LV \tag{17}$$

$$[C_{loss(s)} + C_{cons(s)}] = \sum_i \left( \sum_d \left( N_{A(d)}^{days} \sum_h \rho_{Grid(i,h,d,s)}^{kWh} \times P_{Grid(i,h,d,s)} \right) \right) \tag{18}$$

Where,  $C_{EV}$ ,  $C_{RE}$  and  $C_{ES}$  denote the capital and operating costs for PEV chargers, renewable energy resource units and DESS, respectively,  $P_{RE}$  is the active power provided by renewable energy resources in kW,  $R_{RE}$  is the return of selling energy,  $P_{ES}$  is the active power consumed by the DESS,  $E_{ES}$  denotes capacity of DESS in kWh,  $C_{loss}$  and  $C_{cons}$  represent the costs of energy losses and energy consumed by PEVs, normal load, and DESS, respectively,  $\rho_{Grid(i,h,d,s)}^{kWh}$  is the selling price of renewable energy resources,  $P_{Grid(i,h,d,s)}$  is the energy cost distributed from the grid in \$/kWh,  $N_{A(d)}^{days}$  is the number of actual day,  $P_{Grid}$  is the generated active power from grid,  $LV$  is the levelized cost factor,  $C_{CH}$  is the capital cost of PEV chargers, and  $\chi_{EV}$  is number of charging stations installed at bus  $i$ .

A work in Ref. [77] presented a review of the measurement uncertainty in energy monitoring, where the MCMC method's contributions in this area are elucidated. The authors in Ref. [78] used MCMC method in simulating the wind speed data and implemented an embedded Markov Chain to incorporate the long term effects in modelling the turbulent wind flow, as depicted in Fig. 8. Authors have discovered that the proposed embedded Markov chain outperform the conventional MCMC method.

A slice sampling in MCMC simulation in a case study of probability assessment for power system voltage stability margin with renewable energy source has been presented in Ref. [79]. The slice sampling method performs better than Gibbs sampling method with respect to smaller simulation size, and the calculation efficiency. Besides, the proposed slice sampling method is more efficient and simpler to implement in the power system probabilistic case study. The execution process of the proposed algorithm for power system voltage stability margin using slice sampling in MCMC is illustrated in Fig. 9.

4.2.2. MCMC sampling procedures

Typically, the MCMC sampling is broken down in three main sampling procedures namely; the basic Metropolis – Hastings algorithm, Gibbs sampling algorithm, and Differential Evolution [72]. Each has its own advantages and complexity as well as types of applications. The basic Metropolis – Hastings algorithm is known

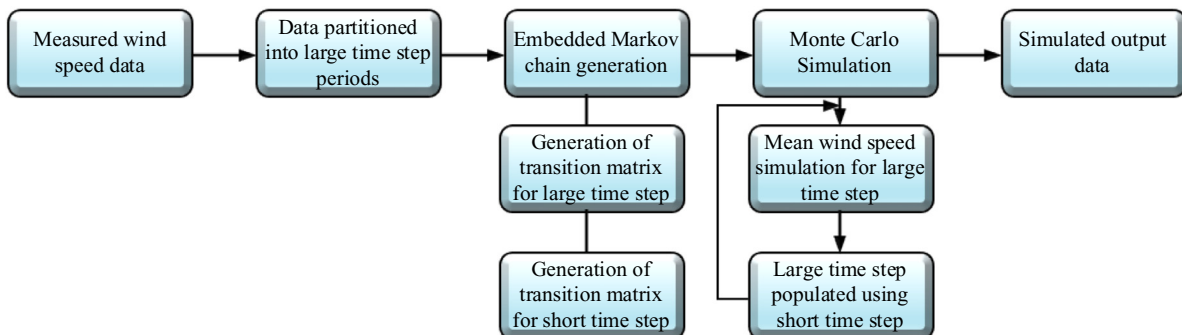


Fig. 8. Markov Chain development and related Monte Carlo simulation [78].

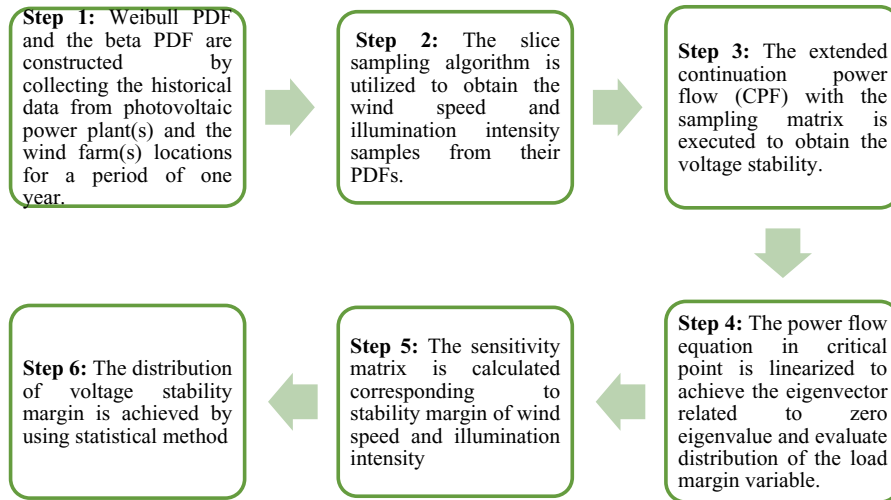


Fig. 9. The voltage stability margin estimation process for power system with renewable energy source using slice sampling.

Table 6  
Main advantages vs disadvantages of main MCMC variants.

References	Type of MCMC	Advantages	Disadvantages
[80]	Metropolis Hastings	<ul style="list-style-type: none"> <li>Knowing the posterior distribution without knowing all the mathematical properties through random sampling despite only knowing the density for different samples</li> <li>Particularly useful in representing posterior distributions that are hard to determine using analytical means</li> <li>Simple implementations for highly correlated distributions</li> </ul>	<ul style="list-style-type: none"> <li>The values calculated must be proportional to the posterior likelihood</li> <li>Only applicable to very strongly correlated parameters</li> <li>Requires a suitable step size to avoid too many rejections from the next sampling sequence or resulting in a poor exploration</li> </ul>
[80]	Gibbs Sampling	<ul style="list-style-type: none"> <li>Produces posterior distribution with good accuracy</li> <li>Easy to evaluate the conditional distributions</li> <li>Conditional distributions will be in lower dimensions and rejection sampling or importance sampling can be applied to these dimensions</li> </ul>	<ul style="list-style-type: none"> <li>Struggles in multi – modal distributions</li> <li>Suffers from Computational efficiency in a long run</li> <li>Suffers from manoeuvrability in cases of strong variables' dependencies</li> </ul>
[81]	Differential Evolution	<ul style="list-style-type: none"> <li>Faster convergence rate in a higher dimension sampling problem</li> <li>Reduction in rejection rate of proposal distributions due to multiple chains of sampling learning from each other</li> <li>Requires simple tuning parameters</li> </ul>	<ul style="list-style-type: none"> <li>Cross – over and exchange between parallel chains of sampling needs to be addressed for better convergence</li> </ul>
[82]	Slice Sampling	<ul style="list-style-type: none"> <li>Does not require much tweakable parameters such as proposal distributions</li> <li>No rejections of samples</li> <li>Suitable when little is known about the sampling distribution</li> </ul>	<ul style="list-style-type: none"> <li>Suffers from curse of dimensionality</li> <li>Sampling is done for each variable in turn using one dimensional sampling in a multi – dimensional distribution</li> </ul>
[83]	Annealing Methods	<ul style="list-style-type: none"> <li>Suitable for sample transitioning from high probability region to another high probability region</li> <li>Does not suffer greatly from curse of dimensionality</li> <li>A heuristic method that is easy to implement</li> </ul>	<ul style="list-style-type: none"> <li>May be developed by trial and error</li> <li>Moving in small steps from one iteration to the next</li> <li>Requires knowledge in tuning its parameters</li> </ul>

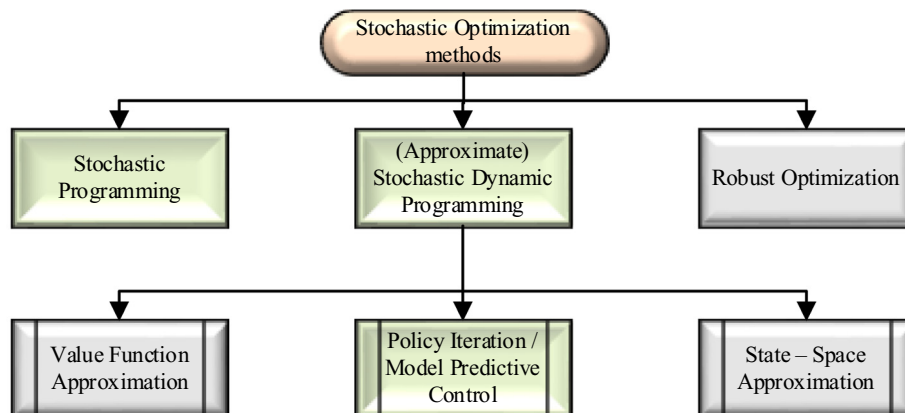


Fig. 10. General overview of stochastic optimization [85].



for its simplicity but lacks the ability to converge properly in problems where parameters are highly correlated. Therefore, a more complex approach would be suitable in a multivariate environment. The Gibbs sampling method by Ref. [80] separates the multivariate problem and treats them independently by sampling from conditional distributions of parameters. The Gibbs method is known for its accuracy but suffers in computational efficiency in a long run. The Differential Evolution sampling procedure is a heuristic method that generates random chained samples that “learn” from each other. Instead of relying on a single random sample and creating a chain from that random sample, multiple random samples with multiple chains are generated using this method. By learning from other chains of samples, the parameter’s correlations between the samples are respected. Hence, it causes the chains of sampling to be formed within the parameter’s correlations limits, leading to a greater efficiency of sampling within the true underlying distribution. However, the Differential Evolution algorithm requires a certain “tuning” parameter to sample efficiently. More information regarding the DE sampling procedure in MCMC can be found in Ref. [81]. Many other main variants of MCMC exist hitherto and is summarized in Table 6. The table highlights the main MCMC sampling variants’ advantages and disadvantages.

**5. Stochastic optimization methods**

As opposed to the deterministic optimization method which assumes a perfect knowledge of a system, the stochastic algorithm models include uncertainties either in predictions, in the decision – making processes, or both. In optimizing the problem formulations under uncertainties in stochastic models, the main approaches are divided in three categories, namely; stochastic programming, robust optimization, and (approximate) stochastic dynamic programming (ASDP) as shown in Fig. 6. The paper’s scope is focused on the renewable energy applications which are in the field of ASDP. Brief information on the stochastic programming methods which are still prevalent in renewable energy applications are shown in the next section. Robust optimization approach is not considered in the paper’s scope. The robust optimization approach generally produced over – conservative results, needed expertise as well as rationale in uncertainty set construction, and difficult to implement in dynamic uncertainty cases [84]. Nonetheless, interested readers are directed to the recent notable works of stochastic robust optimization in renewable energy applications as mentioned in the works of following authors [85–87] (See. Fig. 10).

**5.1. Stochastic programming**

In dealing with power generation problems, stochastic unit commitment in the form of stochastic programming has been implemented as a promising tool [88]. The utilization of scenario – based/tree uncertainty’s representation and probabilities in the optimization is the main idea of the stochastic programming. The stochastic programming models are divided into two – stage models as well as multistage models. The methods were mainly used as stochastic mixed integer programming (SMIP, linear or non – linear SMIP are denoted as SMILP or SMINLP) problems formulations in renewable energy applications.

**5.1.1. Two – stage models**

The former two – stage models separate the optimizations in day – ahead (1st stage) versus real – time (2nd stage) decisions. Typically, in the 1st stage (day – ahead), decisions for conventional generators such as coal power plants and nuclear generators are made beforehand as the start – up and shutdown times for these generators are not immediate. The commitment decision in

operating these conventional generators depends on up/down time requirements of the generators and the various starts up and shutdown costs. Therefore, the uncertainties and quality in forecasting plays a major role in stochastic optimization as it effects the prior decision that must be made.

In the second stage which is the real time operations (i.e. the expected real time operations’ costs), the input variables’ PDF must be known beforehand to generate large number of relevant scenarios relating to the output PDFs. The 2nd stage normally involves the strategy in dispatching renewable resources and reserves (e.g. Pump – hydro storage) over multiple periods of time while considering uncertainties involved. Despite the huge number of scenarios generated in the 2nd stage, the scenarios are not intertwined with each other, implicating that each scenarios outcome is independent of each other. Once the decision has been made for the 1st stage problems, decomposition method is generally used in two – stage models to treat the 2nd stage scenarios independently, resulting in a cluster of much lesser scenarios needed to be optimized. Common decomposition methods used in two – stage programming models are the Benders Decomposition (BD) method [75,76], Lagrangian Relaxation (LR) method [77,78], Bundle methods [92], and Sample Average Approximation (SAA) method [88]. A stochastic two-level model is offered in Ref. [93] to maximize the profit of the EV aggregator in the upper level and minimize the cost paid by the EV owners in the lower level in the competitive electricity markets. The upper level problem relates to the revenue obtained from selling energy to the EV owners and from reducing load based on negative imbalance prices. The upper level problem can be modelled as follows:

$$\begin{aligned}
 & \text{Maximize} \sum_{w \in T} [E_{t,w}^{ch} \lambda_{t,w}^{ch} - E_{t,w}^{DA} \lambda_{t,w}^{DA} - E_{t,w}^{+,B} + E_{t,w}^{+,B} \\
 & + E_{t,w}^{-,B} + E_{t,w}^{-,B}] + \beta \left[ \zeta - \frac{1}{1 - \alpha} \sum_{w \in \Omega} p(w)l(w) \right] \tag{19}
 \end{aligned}$$

Where  $E_{t,w}^{ch}$  is EV demand provided by the aggregator,  $\lambda_{t,w}^{ch}$  is the aggregator selling price  $E_{t,w}^{DA}$  day ahead (DA) EV demand,  $\lambda_{t,w}^{DA}$  is the DA price at time  $t$ ,  $E_{t,w}^{+,B}$ ,  $E_{t,w}^{-,B}$  are the positive/negative energy balance,  $\zeta$  is the rival aggregator scenario index,  $\Omega$  is the number of scenarios with regard to price and demand,  $\alpha$  is confidence level of conditional value at risk (CVaR),  $\beta$  is the risk factor and  $p(w)$  is the probability of occurrence with respect to demand and price and  $l(w)$  denotes the auxiliary variable to control CVaR.

The lower-level problem narrates the decision-making of EV owners and their reaction to the prices which can be expressed as below:

$$\begin{aligned}
 X_{s0}(\zeta) \in \arg \left\{ \text{Minimize}_{X_s(\zeta), Z^{s,s'}} \widehat{E}_t^D \left[ \lambda_{t,w}^{ch} X_{s0}(\zeta) + \sum_{\substack{s \in S \\ s \neq 0}} \lambda_{s,t,\zeta} \right] \right. \\
 \left. + \sum_{\substack{s' \in S \\ s' \neq 0}} \sum_{s' \in S} \widehat{E}_t^D K^{s,s'} Z^{s,s'}(\zeta) \right. \tag{20}
 \end{aligned}$$

Where  $X_{s0}$  is the EV demand percentage delivered by the aggregator,  $E_t$  is the total EV demand,  $\lambda_{s,t,\zeta}$  is the electricity selling prices offered by each rival aggregator,  $K^{s,s'}$  is the cost relates to the reluctance of EV owners for shifting between the aggregators  $Z^{s,s'}(\zeta)$  is the EVs percentage that are shifted between the aggregators.

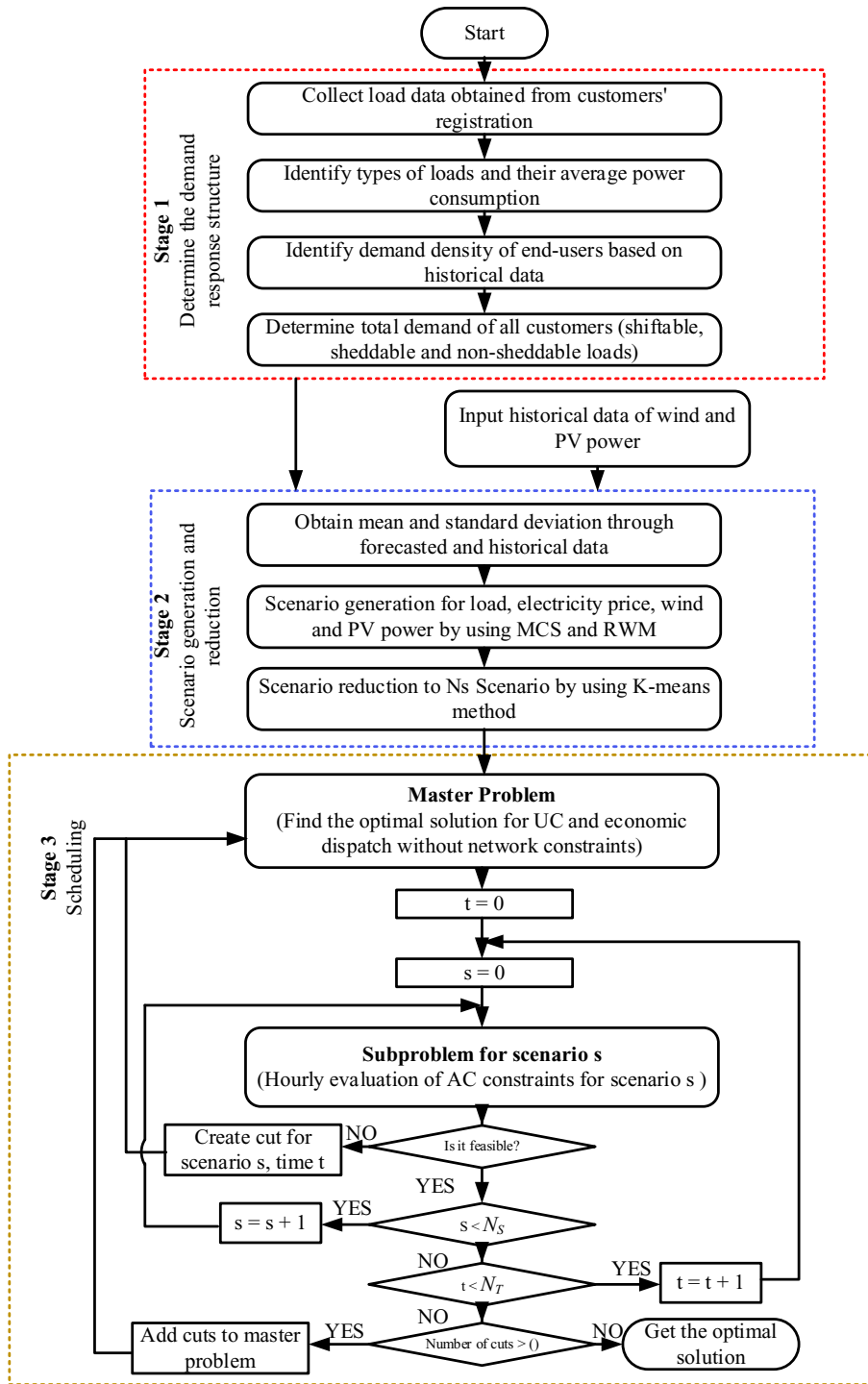


Fig. 11. Implementation framework of stochastic model to solve the optimal scheduling problem in autonomous microgrids [94].

A risk-constrained two-stage stochastic programming is suggested in Ref. [94] to maximize the expected profit during microgrid operator considering uncertainties such as renewable resources, demand load and electricity price. A three-stage efficient flow diagram is developed to represent the underlying the optimal scheduling problem, as shown in Fig. 11. In the first phase, the customers electrical devices and equipment demand are assessed. In the second phase, the scenario generation and reduction process are executed for stochastic parameters. In the third phase, the optimization problem is solved by employing a risk-constraint stochastic programming approach.

The authors in Ref. [95] developed a stochastic model of AC security-constrained unit commitment (AC-SCUC) problem related with demand response (DR) considering uncertainties of wind, PV units and demand-side participation for the day-ahead energy and reserve scheduling in an islanded residential microgrid. In addition to that, an economic model of responsive loads is established based on real-time pricing (RTP) scheme in view of the price elasticity of demand and customers' utility function. The objective function of the proposed model is designed with two terms including the profits associated with here-and-now (H&N) and wait-and-see (W&S) decisions. The objective function includes the purchasing

energy cost from renewable, dispatchable units and buying reserve from DG.

$$\text{Max}(p^{H\&N} + EP^{W\&S}) \quad (21)$$

$$p^{H\&N} = \sum_{t=1}^{N_T} \sum_{j=1}^{N_j} \rho_{j,t} D_{j,t} - \sum_{t=1}^{N_T} \sum_{i=1}^{N_G} [(A_i \cdot u_{i,t} + B_i \cdot P_{i,t}) + SUC_i y_{i,t} + SDC_{i,z,t} (C_{i,t}^{R^D} \cdot R_{i,t}^D + C_{i,t}^{R^U} \cdot R_{i,t}^U + C_{i,t}^{R^{NS}} \cdot R_{i,t}^{NS})] + (C_{j,i}^{R^D} \cdot R_{j,i}^D + C_{j,t}^{R^U} \cdot R_{j,t}^U) \sum_{t=1}^{N_T} \left[ \sum_{w=1}^{N_W} \rho_{w,t} P_{w,t} + \sum_{v=1}^{N_V} \rho_{v,t} P_{v,t} \right] \quad (22)$$

$$EP^{W\&S} = - \sum_{s=1}^{N_S} \sum_{t=1}^{N_T} \sum_{i=1}^{N_G} \pi_s \cdot [SUC_i (y_{i,t,s} - y_{i,t}) + SDC_i (z_{i,t,s} - z_{i,t}) + \rho_{i,t}^{Dep} \cdot (r_{i,t,s}^U + r_{i,t,s}^{NS} - r_{i,t,s}^D)] - \sum_{S=1}^{N_S} \sum_{t=1}^T \left[ \pi_s \cdot \sum_{j=1}^{N_j} \rho_{j,t}^{Dep} \cdot (r_{i,t,s}^U - r_{i,t,s}^D) \right] - \sum_{S=1}^{N_S} \sum_{t=1}^T \left[ \pi_s \cdot \sum_{w=1}^{n_W} \rho_{w,t}^{Dep} \Delta P_{w,t,s} + \sum_{v=1}^{N_V} \rho_{v,t}^{Dep} \Delta P_{v,t,s} \right] - \sum_{j=1}^{N_j} EENS_j \quad (23)$$

In the above equations, the profit of microgrid operator,  $p^{H\&N}$  is assessed using sum of 5 terms. The first one denotes the electricity consumption revenue from customers. The second term denotes the cost associated with distributed generations (DGs) and their start-up/shut-down. The third and the fourth term represents the scheduled reserve cost of generating units and loads, respectively. Finally, the last term represents the energy cost delivered by wind and PV units. Similarly, expected profit of microgrid,  $p^{H\&N}$  is evaluated based on the total of 5 terms. The first one denotes the unit commitment (UC) cost. The second and the third terms represent the deploying reserves cost from DG units and loads, respectively. The fourth term stands for the power cost delivered from wind and PV units in real-time and the day-ahead energy forecasted. Finally, the last term corresponds to the expected energy cost which is not served (EENS). The detail parameter description of the in equations (22)-(23) can be found in Ref. [95].

A work in Ref. [96], proposed a risk constrained two-stage stochastic programming model to determine the optimal scheduling to maximize the expected profit of operator. The flow diagram of the propose framework is operated in two stages, as illustrated in Fig. 12. As it can be observed that, the input data is categorized into two groups, deterministic data and stochastic data. After, a set of scenarios is generated using MG uncertainties. Then, an appropriate scenario-reduction algorithm is employed to reduce the generated scenarios into an optimal subset. In the next stage, the stochastic security and risk-constrained scheduling problems are addressed. The optimal scheduling of the generating units is performed based on unit commitment (UC) algorithm and AC/DC optimal power flow (OPF) procedure by taking into account of objective function and constraints.

Readers are encouraged to read the works specified for each decomposition algorithm, which highlights the past notable implementations of the two-stage methods in power generations and renewable energy applications. Table 7 presents the recent works of two-stage stochastic programming in renewable energy applications.

### 5.1.2. Multi – stage models

In multi – stage stochastic programming models, uncertainties are captured dynamically as possible events branched out of a scenario tree. Each uncertainty in events at a later time  $t+1$ ,

depends on its previous states at time,  $t$ . Decision – making processes are adjusted and updated hourly (or multi – hourly or sub – hourly). Therefore, the multi – stage models represent a more accurate and realistic interactions between decision – makings and

unfolding uncertainties as time goes by. Each scenario generated takes a unique path starting from its root node,  $\mathbf{x1}$  to corresponding end nodes (i.e.,  $\mathbf{x6}$ ,  $\mathbf{x8}$ , and  $\mathbf{x15}$ ), where each node along the path represents the time at which decisions were made. For each corresponding scenario,  $\mathbf{n}$  (i.e.  $\mathbf{n1}$  taking the node from  $\mathbf{x1} \rightarrow \mathbf{x2} \rightarrow \mathbf{x3} \rightarrow \mathbf{x5}$ ), the problem is treated as an individual deterministic problem. The difficulty of the multi – stage models rises from the non – anticipative constraints, which means that only one set of decision variables are permissible at each node. The advantages of the multi – stage models come with a huge computational expense. The number of scenarios grows exponentially as shown in Fig. 13. Hence, multi – stage models are harder to solve than the two – stage models. Advanced decomposition models/algorithms are typically introduced in these cases. Often, the techniques used are nested or multi – layered decompositions and are further divided into scenario – based decomposition and unit – based decomposition targets [6]. Common advanced decomposition algorithms in multi – stage stochastic programming and its past notable works are shown as follows; Augmented LR [105], Dantzig – Wolfe decomposition (Column Generation (CG)) [106], Progressive Hedging [107], Nested CG [108], Stabilized LR or CG [109].

The algorithms summarized in Table 9 are used in the past notable works of multi – stage – stochastic programming. In Table 9, readers are also enlightened with the qualitative advantages and disadvantages of the highlighted algorithms in multi – stage stochastic programming, while Table 8 presents the recent works of multi – stage stochastic programming in renewable energy applications. Quantitative comparisons of the two – stage and multi – stage models can be found in the past works of [110,111]. Qualitative advantages and disadvantages of these methods are summarized in Table 9.

From the literatures surveyed based on Table 7 & Table 8 in renewable energy applications, it is apparent that the two – stage stochastic models are preferably implemented due to its simplicity in implementations and a guaranteed convergence in obtaining the solution. However, the multi – stage stochastic models are becoming more reliable as it better represents the complexity of renewable systems with significant increase in renewable resources and storages. Various advanced decomposition in two – stage and multi – stage models have proven to yield better results than the deterministic as well as perfect foresight cases (i.e. [101,114]). The literatures in two – stage stochastic models provided a rather conservative solution with respect to multi – stage

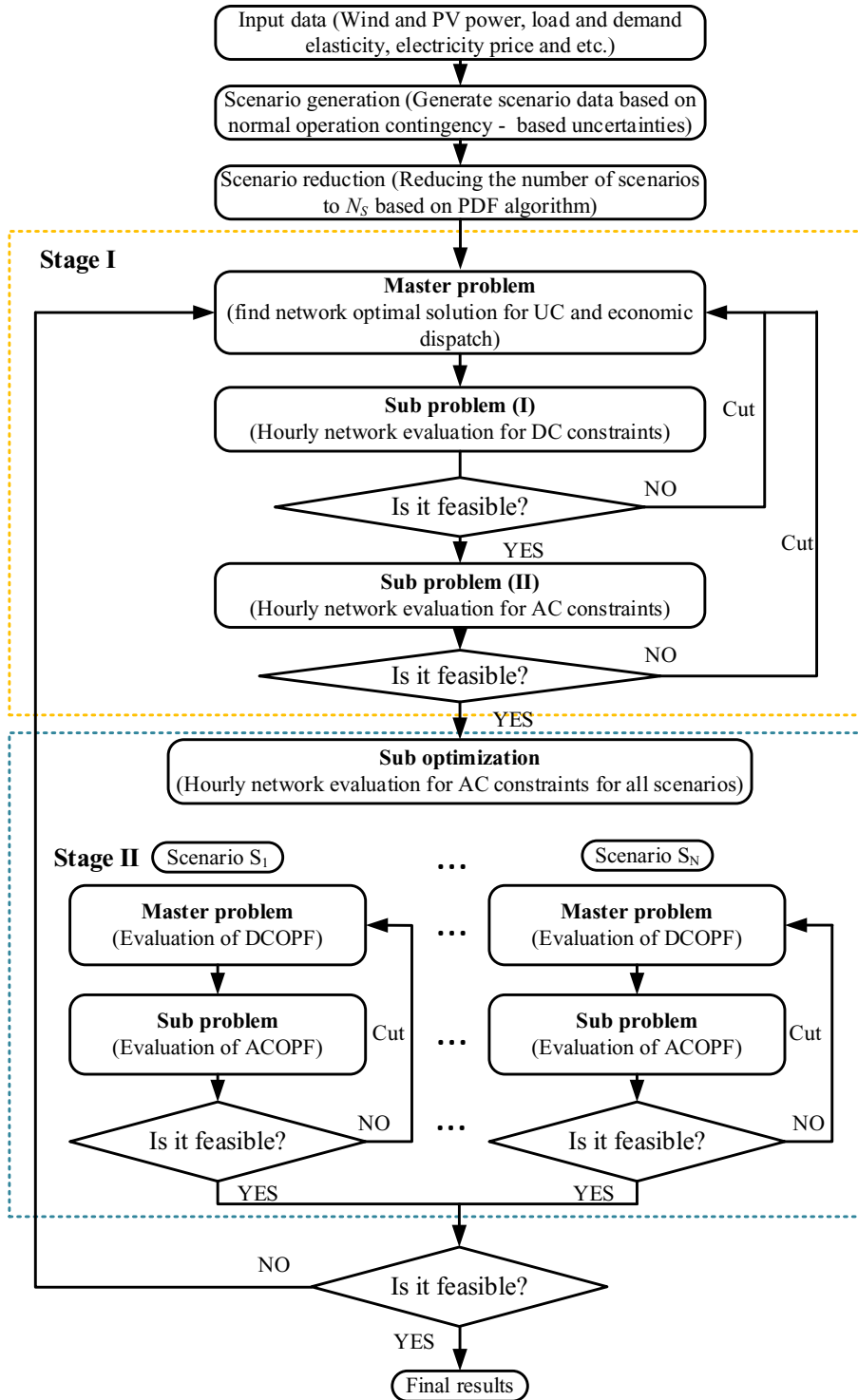


Fig. 12. Methodological framework of a stochastic model for energy and reserve scheduling considering risk management strategy [96].

stochastic models that may lead to inefficiencies in generating the best solutions. With advancements of computational efficacies, multi – stage stochastic models are becoming more viable in solving stochastic renewable energy problems. Applicability of multi – stage stochastic models (short – term and long – term) especially in big – scaled renewable economic dispatch are yet to be fully explored. Demand side uncertainty and considering demand side response has been gaining a lot of attentions in formulating the stochastic renewable systems' problems. Many recent

literatures on stochastic programming (i.e. [82,83,88,89]) have started to consider the demand side uncertainty and managing the demand side in optimizing the renewable's system. Main advantages of a responsive demand side management are the reduction in costs and minimization of energy wastage. It is to be noted that literatures combining the stochastic programming methods with meta–heuristic algorithms were not being considered in this section and only SMIP method variations were highlighted.

**Table 7**  
Two – stage stochastic programming methods in renewable energy applications.

References	Methods	Structures	Objective(s)	System's uncertainty	Main Result(s)	Future work(s)
[97]	SMILP	Two – stage	Minimize daily operational costs	Wind power and Energy storage	The proposed stochastic methods reduced the total daily costs and load shedding	N/A
[98]	Multi – objective SMILP	Two – stage	Minimize operational cost and pollution	Demand side, supply side (renewable), and energy storage	Applying portable renewable energy resources have decreased the cost and increased profits	N/A
[99]	Novel decomposition – SMILP	Two – stage	Minimize NPV of total expected costs	Solar irradiance, wind, and load	The proposed method demonstrated the effectiveness in finding the optimal battery energy storage system (BESS) power and energy sizes	Envisaged to be used in MG planners, Govt. and private agencies
[100]	BD – SMIP	Two – stage	Minimize day ahead purchase cost and expected resource cost	Demand side, supply side (RE), electricity prices	Day – ahead power procurement and the formulation as a two – stage SMIP problem is proposed	Demand–side procurement by two–stage stochastic am
[101]	BD – SMINLP	Two – stage	Minimize expected total operation costs including generation, day – ahead market, and battery wear for the next 24 h	Demand, EV availability and storage capacity, renewable energy resources	Using the energy from EV reduces the total operation cost of the microgrid. The results yielded better cost savings than a deterministic model	N/A
[102]	BD – SMILP	Two – stage	Minimize environmental and social impacts	Wind speed, solar irradiation, and demand	Including demand response as a flexible load reduces load curtailment and reduces energy generation needed	N/A
[103]	$\epsilon$ – Constraint multi – objective SMILP	Two – stage	Maximize DG owners' profits and minimize Distribution Company's (DisCo) costs	Wind speed, load, electricity price	Solving the reconfiguration of the network and allocation of DG simultaneously produced a more desired scheduling between the stakeholders. The stochastic optimization is compared to a deterministic optimization with an improved profit on behalf of the DG owners	N/A
[104]	Scenario – based SMINLP	Two – stage	Minimize active and reactive power purchasing costs	Load demand, wind power	Reduction of expected costs of energy and reactive power as well as emission costs	N/A

5.2. Approximate stochastic dynamic programming

Stochastic dynamic programming is an optimization method in solving discrete multi – stage decision – making processes with underlying uncertainties or probabilities. Decisions made to lower the objective function's costs at a current stage might

unintentionally increase the total costs throughout the whole period of optimizations. One need to evaluate decisions made at all stages carefully to obtain the most cost – efficient objective function. Stochastic dynamic programming method can capture the trade – off between decisions made in the present and the future stages. Due to these properties, it is instinctive that stochastic

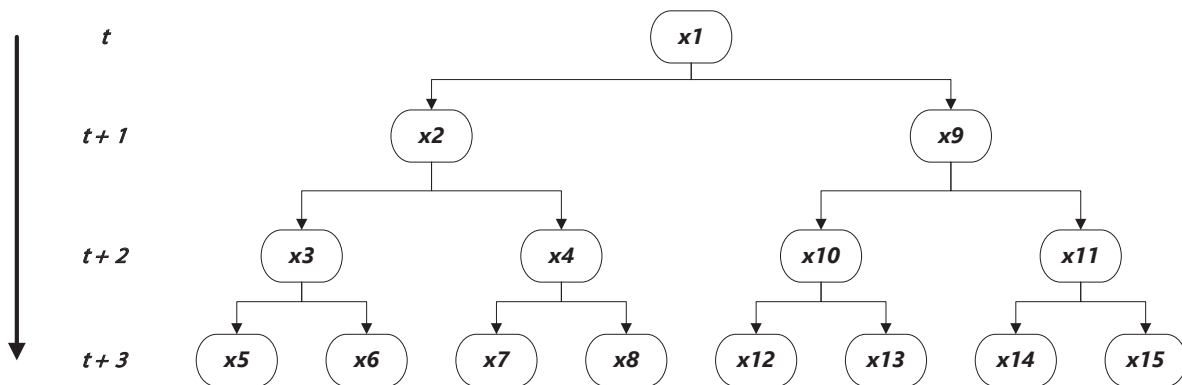


Fig. 13. Scenario tree with multiple stages (4 stages, 8 scenarios, and 15 nodes).



**Table 8**  
Multi – stage stochastic programming methods in renewable energy applications.

References	Methods	Structures	Objective(s)	System's uncertainty	Main Result(s)	Future work(s)
[112]	Dynamic Response – SMILP	Multi – stage	Maximize net social benefit	Demand side and Energy Storage	A responsive demand side provided a more flexible and smarter power systems	Enhancing planning methodologies using k – means and system states
[113]	Decision dependent – SMILP	Multi – stage	Maximize total profit	Wind capacity penetrations and demand	The proposed method provided effective optimization information on investment and long – term expansion planning	Developing new models with uncertainties constraints
[114]	Two – period multi – stage SMILP	Multi - stage	Minimize NPV related to losses, emission, maintenance, operation, and unserved energy	Generation sources, electricity demand, emission prices, demand growth	The proposed method produced significantly better results in terms of objectives and yielded robust decision - makings in comparison to deterministic methods	N/A
[115]	Piecewise multi – stage linear stochastic optimization	Multi - stage	Minimize operational costs and computational time of long – term generation scheduling of hydropower	Load and Water inflow	Inclusion of piecewise linear approximation boosted the computational efficacy and minimized the operational costs in operating large storage capacity hydro power plants	N/A

**Table 9**  
Qualitative advantages and disadvantages of the two – stage and multi – stage stochastic programming algorithms.

Stochastic Optimization Method	Type	References and Algorithms	Advantages	Disadvantages
Stochastic Programming	Two – stage	[80,81] Lagrangian Relaxation (LR) [88] Sample Average Approximation (SAA) [78,79] Benders Decomposition [92] Bundle Methods	<ul style="list-style-type: none"> <li>• Simple Implementations and easier to understand</li> <li>• Convergence and good performances are guaranteed as various decomposition methods have been tested</li> <li>• Robustness issues can be addressed using risk measurements</li> <li>• Value of stochastic solution and expected value of perfect information can be provided</li> </ul>	<ul style="list-style-type: none"> <li>• Probabilities of scenario generated need to be known</li> <li>• Computationally expensive for large number of scenarios generated</li> <li>• Complexity in dealing with integer variables during the 2nd stage (i.e. unit rescheduling in real – time)</li> <li>• Assumption of static uncertainties</li> </ul>
	Multi – stage	[105] Augmented LR [106] Column Generation (CG) [107] Progressive Hedging [109] Stabilized LR or CG	<ul style="list-style-type: none"> <li>• Considering over multiple time intervals of uncertainties in decision – making processes</li> <li>• Uncertainties and decisions can be adjusted dynamically as scenarios unfold</li> <li>• Advantageous for systems that needs quick rescheduling</li> <li>• Providing value of perfect information and value of stochastic solution</li> </ul>	<ul style="list-style-type: none"> <li>• Size of problems grows exponentially with increasing time intervals and scenarios</li> <li>• Requires explicit scenario tree representations</li> <li>• Difficulties increase with integer variables present in all stages</li> </ul>

dynamic programming is suitable in the applications of renewable energy optimizations.

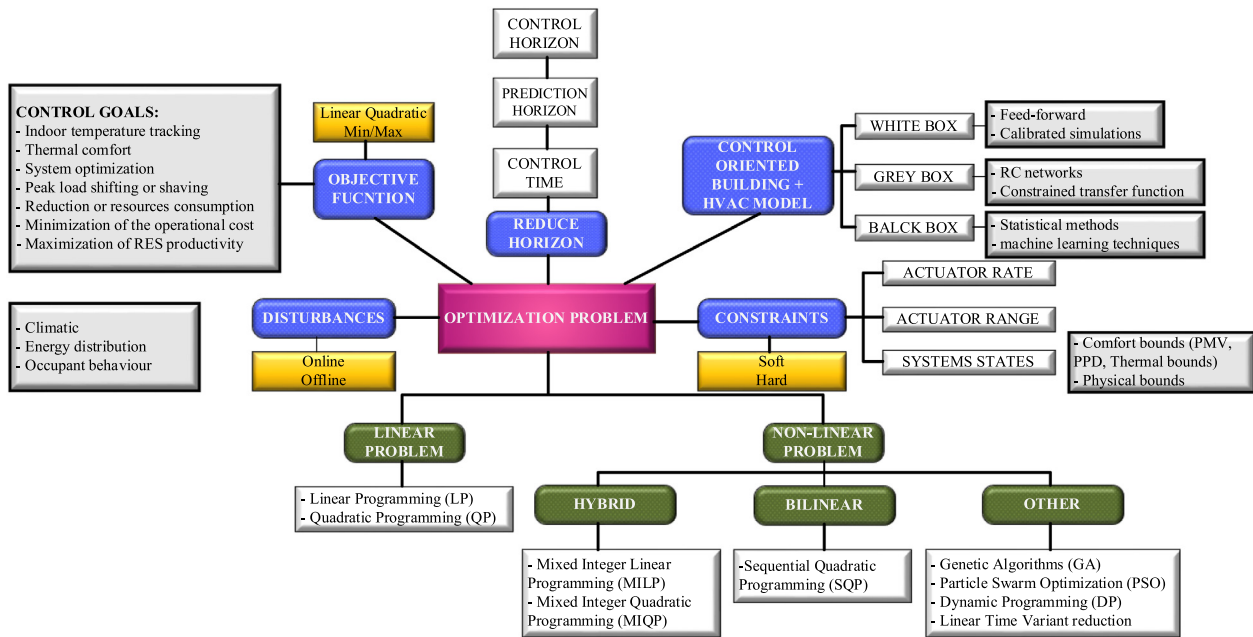
The usage of dynamic programming can be dated back to late 1970s [90] in solving deterministic problem. The solving approach was based on Bellman's Principle of Optimality [91] which uses the backward induction method. The past works of dynamic programming suffer from heavy computational expenses due to the curse of dimensionality. As the number of scenarios and states increases as stages unfolds, the time needed in yielding a solution grows exponentially. Hitherto, various methods and broad class of algorithms have been tested to overcome the computational expenses.

Approximate stochastic dynamic programming (ASDP) has proven to lighten the burden of dimensionality's curse of dynamic programming and is well suited for models with uncertainties and stochasticity [116]. Generally, the ASDP method can be divided in

three categories as shown in Fig. 6. The scope of the paper is within the policy function approximation in the form of stochastic Model Predictive Control (MPC) in renewable energy applications. Readers are directed to the recent renewable energy applications mentioned which highlights the usage of value function approximation [117–119] and state – space approximation [93,94] methods. A comparison of approximate dynamic programming techniques was carried out by Ref. [122]. Authors have compared various policy iteration and value function approximation techniques. Authors have argued that new theory and methodology are needed for these techniques in order to solve real – world problems, which are becoming more difficult.

5.2.1. Model predictive control (MPC)

MPC, also known as receding control horizon approximates policies by iteratively solving a finite horizon optimal control



**Fig. 14.** Framework of the MPC optimization-based heating, ventilation, and air conditioning (HVAC) systems. The boxes highlighted with blue denote the factors that have impact to the optimization problem directly; the boxes highlighted with green indicate the optimization problems results [123].

problem. The horizon recedes once the optimal control for a current stage,  $t$  has been found moving on to another finite horizon at a later stage,  $t + 1$ . The process is repeated until the optimal control has been found for all stages;  $t$  (initial stage) until  $t_{max}$  (final stage). Serale et al. [123] have suggested several parameters which have a direct impact to the MPC optimization problems namely; Objective function, receding horizon, control model, constraints, and disturbances, where the optimization problems of MPC can be further divided into Linear and Non – Linear problem formulations. The framework of the proposed method is illustrated in Fig. 14.

Many works in the scope of MPC have been found in literatures and are highlighted in the next paragraph. Recent literatures in stochastic MPC are mentioned in the later paragraphs of this section and summarized in Table 10, Table 11, and Table 12.

### 5.2.2. Notable overview of MPC

Several recent reviews in MPC have been published in the lights of power generations, building and environments, and renewable energy applications. Interested readers are encouraged to read the works of following authors where theoretical modelling and applications of MPC are further discussed. The state – of – the – art development of MPC has been reviewed by Ref. [124] for renewable energy applications. The authors have presented a systematic review of MPC applications in the field of solar PV and wind energy renewable systems. The authors aimed to help researchers in further exploring the flexibility of MPC for design, implementation, and analysis in renewable energy applications. In Ref. [125], hierarchical energy management strategy based on model predictive control is proposed for microgrid management operation considering different endogenous and exogenous sources of uncertainties. In Ref. [123], MPC in the themes of enhancing building and HVAC system energy efficiency have been systematically reviewed. The potential benefits and future of MPC are discoursed at a great depth in authors' work. ANNs based MPC has been reviewed by Ref. [126] in a case study of a residential HVAC system. The authors have utilized new ANNs algorithm to design a supervisory MPC which successfully reduced operating costs of equipment while constraints are not violated with a cost reduction percentage range of 6%–73% depending on the season. A similar review was made by

the main author in his past work [127] with regards to theory and applications of HVAC control systems using MPC and was regarded as the most remarkable review on MPC due to clear classifications and comprehensive scheme of MPC implementations.

### 5.2.3. Stochastic MPC implementations in renewable energy

Stochasticity of MPC in recent renewable energy applications are typically represented as probability – constrained scenarios or forecasts, uncertainty modelling of scenario generations, and random disturbances. Stochastic MPC can be further derived into three main categories which are tree – based, chance – constrained, and multi – scenario MPC. The summary of recent literatures pertaining to these categories is mentioned in the next sections.

#### 5.2.3.1. Tree – based MPC implementations in renewable energy.

Tree – based MPC works with an assumption of time dependant events can be synthesized from a rooted tree, where the most relevant possible disturbances can be captured. The concept of tree – based MPC is quite similar to the multi – stage stochastic programming approach (refer Fig. 7). Each root to different nodes' paths represents a possible disturbance scenario, where the branching of the paths symbolizes the different forecast possibilities and uncertainties along a given prediction horizon. Each node at a given point in time,  $t$  corresponds to a control action that can be taken at that time. One must note that the control action taken must not be allowed to diverge before the bifurcation points. The tree – based MPC utilizes the ensembles of forecasts and solves it by considering the sequences contained in the tree. Different paths/branches of the tree nodes are treated as individual deterministic problems. The path with the least costs or the most efficient in terms of given objective functions are implemented at current time,  $t$  as a control action. The process is repeated until the control optimizations over the entire horizons have been obtained.

A hybrid robust and stochastic accelerated MPC have been implemented in the work of [128] with 24 h horizon window for EV integrated microgrid energy management considering demand response. The authors have utilized the hybrid MPC with forecasts coupled with Benders decomposition (BD) method to

**Table 10**

Tree – based stochastic MPC in renewable energy applications.

References	Method	Objective	MPC Type	Control horizon	Sampling resolution	System's uncertainty	Main Results	Future Work
[128]	Stochastic accelerated MPC	Minimize total daily operational costs	Tree – based	16 h	½ hour	EV charging demand, load, real – time electricity price, renewable energy output	The stochastic MPC outperforms the deterministic MPC by lower total daily operational cost in all cases	Extending the proposed method with available EV charging load prediction models
[129]	Risk – averse stochastic MPC	Maximize profit and minimize risks (CVaR)	Tree – based	24 h	1 h	Wind power forecasts, price of energy	The proposed method outperforms all mentioned methods and marginally expected profit compared to perfect solution	Application of the proposed method to real – world cases and other renewable applications
[130]	CVaR fault tolerant stochastic MPC	Optimize CVaR	Tree – based	4 steps ahead	1 s	Wind power forecasts	The proposed method has achieved a control performance of 40% higher than the common Min – Max performance index	Solving the proposed stochastic MPC in one step to yield a higher practical value

**Table 11**

Chance - constrained stochastic MPC in renewable energy applications.

References	Method	Objective	Type of MPC	Control horizon	Sampling resolution	System's uncertainty	Main Results	Future Work
[131]	Multi – time scale stochastic – heuristic MPC	Minimize weekly operational costs	Chance – constraint	12 h	5 min/1 h	PV power forecast, plug in EV, deferrable and non – deferrable appliances in smart home	Shifting the hour–scale and day–scale appliances to the optimal hours and week of the day can substantially reduce the weekly operational costs	Applying the proposed method in multi – scale microgrids
[132]	Stochastic two – stage MPC	Minimize cost of energy and emissions of greenhouse gases	Chance – constraint	6 h	1 h	Renewable energy resources, demand	Experimental results have proven the feasibility and implementation ability of proposed stochastic MPC that outperforms the deterministic MPC	Analysing the scalability of proposed framework and investigating distributed methods
[133]	Stochastic warping function MPC	Minimize wind power tracking error	Chance – constraint	1–12 h	5 min	Wind power forecasts	The proposed stochastic MPC outperforms the deterministic MPC in power tracking errors	The proposed control system can be integrated into currently existing system
[134]	Stochastic – EMPC	Minimize microgrids' operating costs	Chance – constraint	72 h	1 h	Renewable supply, Load demand	The proposed method achieved a better trade – off between performance and computational efficacies in comparison to centralized scheme	Incorporating the topology of distribution network, energy exchange between MG and fluctuating prices of energy
[135]	Stochastic MPC	Minimize operational costs	Chance – constraint	24 h	½ hour	RE generations, load, demands, EV, and electricity prices	The stochastic MPC framework outperforms the traditional day – ahead programming strategy in terms of minimizing the operational costs	Applying the stochastic MPC in a multi scale microgrid systems

simultaneously reduce total operational cost in energy management as well as improve computational efficiency. Simulation results showed that the proposed method outperforms the standard deterministic MPC method with regard to total operational cost by a margin of around 10%. The algorithm of stochastic BD applied to MPC is shown below.

---

**Algorithm:** Stochastic BD

---

At each time step  $t$  **do**

**Initialize**  $\mu = 1$ , Upper Bound (UB) =  $+\infty$ , Lower Bound (LB) =  $-\infty$ ,  $\theta = 0$

**For**  $k = 1$  to  $N$

**Set**  $\mu = 0$ , **do**

Solve master problem in Equation (24) and determine a trial solution  $X_{\mu,g,t+k|t}$

Update the value of LB

Solve all sub-problems in Equation (25) with trial solution  $X_{\mu,g,t+k|t}$

Update the value for UB

**if**  $LB/UB > \delta$  **then**

Add the new optimally cut  $\theta$  associated with iteration  $\mu$  to the master problem

Set  $\mu = \mu + 1$ , and repeat the solution procedures

**else**

an optimal solution is obtained, and implement the control signals at time step  $t$

**end for**

---

$$Z_\mu = \min \sum_{k=0}^N \left[ \lambda_f G_{g,t+k|t} X_{g,t+k|t} + F(X_{g,t+k-1|t}, X_{g,t+k|t}) + \theta_k \right]$$

$$\theta_\mu \geq \sum_p \pi_{p,t+k|t} \sum_w \pi_{w,t+k|t} \left[ Q_{\mu,p,w,t+k|t} - \varepsilon_{\mu,p,w} G_{g,t+k|t} \right. \\ \left. (X_{\mu,g,t+k|t} - \bar{X}_{\mu,g,t+k|t}) \right] \quad (24)$$

$$Q_{\mu,p,w,t+k|t} = \min \left( \lambda_{ex,t+k|t}^p E_{ex,t+k|t}^{p,r} + \lambda_f G_{l,t+k|t}^{p,r} \right) \quad (25)$$

Where  $\lambda_f$  is fuel price,  $G_{g,t}$  is the gas input of combined heat and power (CHP) units,  $F(X_{g,t+k-1|t}, X_{g,t+k|t})$  is penalty function used to control the frequency changes during the on/off operating state,  $\theta_\mu$  is Benders cut at iteration  $\mu$ ,  $Q_{\mu,p,w,t+k|t}$  denote the sub-problems value at iteration  $\mu$  under  $p$ th and  $w$ th scenario,  $X_{\mu,g,t+k|t}$  is the

trial solution at iteration  $\mu$ ,  $\varepsilon_{\mu,p,w}$  represents the sensitivity for the corresponding  $Q_{\mu,p,w,t+k|t}$ , and  $E_{ex,t}$  indicates the energy purchased from the external grid.

A risk – averse stochastic MPC based on real – time operation has been developed by Ref. [129] for a wind energy generation system combined with a pumped hydro storage unit to maximize

profit and minimize risks in day – ahead bidding strategies. Authors have compared the results of stochastic MPC method with several other methods such as deterministic MPC, bid – following heuristic and open – loop algorithms. The stochastic MPC method outperforms all other methods and reached an expected profit close to the perfect information solution with a margin of around 2%. Fault tolerant control problem of wind energy conversion systems have been addressed by Ref. [130] using stochastic MPC based on CVaR. Authors have implemented the Markov jump linear model to model randomness of the wind energy conversion systems. A scenario – tree is created within the prediction horizon to transform the stochastic MPC problem to a deterministic MPC. The method produced a better fault tolerant control performance in comparison to the Min – Max performance index. The objective function formulation of CVaR using SMPC algorithm is shown below.

---

**Algorithm:** SMPC algorithm of CVaR objective function

---

**1. Prepare the controller  $C_i$**

1.1 Generate  $m$  scenario trees according to different root node  $w(k) = i (i \in w(k) = \{1, 2, \dots, m\})$

1.2. Calculate corresponding Controllers  $C_i$

**2. Estimate VaR**

set  $\beta = 90\%$

Solve SMPC problem in Equation (26)

The VaR is given by Equations (27) and (28)

**3. Estimate SMPC of CVaR**

**For**  $i=1:3$

$CS\{\cdot\}.f = [ones(1, s * nu), \pi_1, \pi_2, \dots, \pi_s]$ , % decision vectors weights

Calculate other parameters;

**end for**

set  $T$ ; %simulation time

**for**  $k=1:T$

measure  $x(k)$

solve CVaR SMPC problem in Equation (29) and obtain  $u_i$

apply  $u(k) = u_i$

**end for**

---

$$\min_u \sum_{i \in T|T_1 \cup S} \pi(x_i - x_r)^T Q(x_i - x_r) + \sum_{i \in T|S} \pi_i u_i^T R u_i$$

$$s.t. \begin{cases} x_1 = x(k) \\ x_i = A(w(k))x_{pre(i)} + B(w(k))u_{pre(i)} + D(w(k)) + D_1(w(k))e(k) + Iw*yr(k), i \in T|T_1 \\ G_x x(k) + G_u u(k) \leq g, k = 0, \dots, N, \forall w(k) \in W \end{cases} \tag{26}$$

$$G_{xj}, G_{vj}, G_{dj}, G_{d1j}, G_{d3j} \tag{27}$$

$$\alpha_{\beta(u)} = \min\{\alpha \in \mathbb{R} : \psi(u, \alpha) \geq \beta\} \tag{28}$$

$$\min_{u, \{\pi_j\}_{j=1}^s} [\pi_1 \pi_2 \dots \pi_s] \mu$$

$$s.t. \begin{cases} \mu \geq G_x x(k) + G_u U(k) + G_d Iw1 + G_{d1}(Iw1*e(k)) + G_{d3}*(Iw3*yr(k)) - L(r + a) \\ \mu \geq G_x x(k) - G_r U(k) - G_d Iw1 + G_{d1}(Iw1*e(k)) - G_{d3}*(Iw3*yr(k)) + L(r + a) \\ \mu \geq 0, j = 1, \dots, s \\ U(k) \leq \bar{u} \\ U(k) \geq \underline{u} \end{cases} \tag{29}$$

Where the  $x_k$  is the present state,  $\pi_i$  represents the realization probability of scenario  $i$ ,  $Q$  and  $R$  denote weight matrixes,  $G_x \in R^{n_x+n_u}$  and  $G_u \in R^{n_x+n_u}$  stand for coefficient matrixes used in state and input constraints,  $f(u, w)$  is the estimation error,  $\beta$  is the probability factor. By using the probability density of error,  $\alpha$  can be found as  $\alpha = \beta - VaR$ . The jumping information in Markov jump linear model is denoted by  $G_{xj}, G_{vj}, G_{dj}, G_{d1j}, G_{d3j}$ .

5.2.3.2. *Chance – constrained MPC implementations in renewable energy.* Chance – constrained MPC relies on the formulation of output constraints with a given type  $y_{min} \leq y \leq y_{max}$  as chance constraints as shown below:

$$Pr(y_{min} \leq y \leq y_{max}) \geq \lambda \tag{30a}$$

where  $Pr(x)$  is the probability of an event  $X$  occurring,  $y$  is the

**Table 12**  
Multi – scenario stochastic MPC optimizations in renewable energy applications.

References	Method	Objective	MPC Type	Control horizon	Sampling resolution	System's uncertainty	Main Results	Future Work
[136]	Adaptive constrained stochastic MPC	Minimize operation costs	Multi - scenario	1–24 h	0.01–1 s	Renewable energy sources, electrical loads	The method produced a less conservative solution compared to the robust MPC approach	N/A
[137]	Various stochastic MPC	Compare multiple types of MPC	All types	5 steps ahead	30 s	Renewable resources, Load, hydrogen – based PEM electrolyser and fuel cells, lead acid batteries' state of charge	Chance – constrained MPC outperforms other MPC types resulting in a lower cost and less energy exchange in a hydrogen based microgrid	N/A

output of a given process within the constraints of  $y_{min}$  &  $y_{max}$ , and  $\lambda$  is the confidence level of such constraints that can be satisfied.

According to *equation (2)*, the basic idea of a chance – constrained MPC is to solve the optimization problem in each horizon while guarantying the satisfaction of the constraints with a certain probability. It is to be noted that the chance – constrained MPC involves the careful selection of future output predictions and its

uncertainties. Since exact future output predictions can't possibly be captured, uncertainties are represented in either of these two ways; which is either the uncertainty in future disturbances or uncertainty of the process outputs due to manipulated variables. Within this realm of solving probabilities and uncertainties in chance – constrained MPC, several recent publications have been identified and listed below.

A multi – time scale stochastic MPC combined with genetic algorithm (GA) is proposed in Ref. [131] in order to perform scheduling deferrable appliances and energy resources of a smart home (SH) system. The stochastic parameters namely; solar irradiances and its prediction uncertainties are forecasted using neural network toolbox in MATLAB. The uncertainties of the appliances' usage as well as the economic and technical constraints of other energy sources such as diesel generators, batteries, and PV panels are also modelled by the author. The objective function is developed for SH with a goal to minimize the value of the stochastic forward-looking objective function subject to various constraints. A



total of 6 cost terms are taken into consideration to develop the objective function including fuel cost  $C_{d,t}^{F.DG}$ , carbon emissions cost  $C_{d,t}^{E.DG}$ , start up cost  $C_{d,t}^{STU.DG}$  shut down cost  $C_{d,t}^{SHD.DG}$  of DG, switching price of PEV battery  $C_{d,t}^{SW.PEV}$  and cost or income due to the power distribution with the grid  $P_{d,t}^{Grid} \times \pi_{d,t}^{DISCO}$

$$\min_{P_{d,t}} F_{d,t}^{FL} = \min_{P_{d,t}} \sum_{PV} F_{d,t}^{FL} \times Q_{d,t}^{PV}, d \in D, t \in \{t_1, t_2\}, \forall t_1 \in T_1, \forall t_2 \in T_2 \quad (30b)$$

$$F_{d,t} = \left\{ \begin{array}{l} [C_{d,t}^{F.DG}] + [C_{d,t}^{E.DG}] \\ + [(1 - x_{d,t-1}^{DG}) \times x_{d,t-1}^{DG} \times C_{d,t-1}^{STU.DG}] \\ + [x_{d,t-1}^{DG} \times (1 - x_{d,t-1}^{DG}) \times C_{d,t-1}^{SHD.DG}] \\ + [x_{d,t}^{PEV} \times C_{d,t}^{SW.PEV}] + [P_{d,t}^{Grid} \times \pi_{d,t}^{DISCO}] \end{array} \right\} d \in D, t \in \{t_1, t_2\}, \forall t_1 \in T_1, \forall t_2 \in T \quad (31)$$

The multi – time scale MPC divided the control optimization in scale of minutes and hours in a weekly operation, where usage of certain appliances is dominant in their respective time scale. The author has shown that the proposed MPC method has managed to notably decrease the weekly operational cost of the smart home system.

An experimental case study was conducted by Ref. [132] in the operation management of microgrids using stochastic MPC to optimize the economic and environmental parameters. Uncertainties due to renewable energy resources and demand were considered and the stochastic optimization was solved by using mixed – integer linear programming toolbox via commercial solvers. Experimental results have proven the feasibility and implementation ability of stochastic MPC that outperforms the deterministic MPC. Kou et al. [133] proposed a stochastic MPC for wind farm energy dispatch strategy with BESS using probabilistic wind power forecasts. The method considers the non – gaussian wind power forecast uncertainties using chance – constraints warping function. The authors have shown that the proposed method outperforms the deterministic MPC method in terms of power tracking errors while maintaining the state of charge (SOC) of the battery within operational limits. The chance constraint optimization problem is developed to enhance the wind power dispatchability and lessen its oscillation, as shown in equation (32). In addition, SOC constraints and charge discharge power constraints are assigned in order to protect the battery from being overcharged and over discharged, as expressed in equation (33) and equation (34) respectively.

$$\min_{u(k+h|k), e(k+h|k)} \bar{J} = \beta \sum_{k=1}^H e(k+h|k)^2 + (1 - \beta) \sum_{h=0}^{H-1} u(k+h|k)^2, \text{ Subject to} \quad (32)$$

$$Pr \left[ |y(t+k|t) - y_{ref}(t+k)| \leq e(t+k|t) \right] \geq \alpha, h = 1, 2, \dots, H$$

$$SOC_{\min} \leq x_2(k+h|k) \leq SOC_{\max}, h = 1, 2, \dots, H \quad (33)$$

$$-P_{B,\max} \leq u(k+h|k) \leq P_{B,\max}, h = 1, 2, \dots, H \quad (34)$$

Where  $e(k+h|k)$  is the set of auxiliary variables,  $y(t+k|t)$  is the stochastic variables,  $y_{ref}$  denotes the reference,  $Pr$  is the probability,

$\alpha$  and  $\beta$  denote the confidence and trade-off parameter respectively.

A new distributed chance – constraints stochastic EMPC scheme has been presented in Ref. [134] for coordinated stochastic multiple microgrids energy management. The supply and demand side uncertainties were handled using the probabilistic forecasts of renewable generations and load of each microgrid that is in a cooperation scheme with each other. The proposed method successfully reduced the system operating costs and achieved the supply and demand balance in each microgrid within the control horizons through the development of distributed network operator (DNO) controller. DNO acts as an intermediary between microgrids, thus the energy selling between any two microgrids is performed indirectly through DNO. The mathematical expression of cost function is presented in the following equation,

$$\min_{E_{pur,D}^{(k+h|k)}, E_{sel,D}^{(k+h|k)}, E_{pur,m}^{(k+h|k)}, E_{sel,m}^{(k+h|k)}} J_D = \sum_{h=0}^{H_D-1} \left( E_{pur,D}(k+h|k) \eta_{pur,D} + \sum_{m=1}^M E_{sel,m}(k+h|k) \eta_{sel,m} - E_{sel,D}(k+h|k) \eta_{sel,D} - \sum_{m=1}^M E_{pur,m}(k+h|k) \eta_{pur,m} \right) \quad (35)$$

Where,  $E_{pur,D}$  and  $E_{pur,m}$  denote the energy purchased from the grid and DNO respectively,  $E_{sel,D}$  and  $E_{sel,m}$  represent the energy sold back to the main grid and DNO respectively.  $\eta_{pur,D}$  and  $\eta_{pur,m}$  stand for energy price purchasing from the grid and DNO respectively while  $\eta_{sel,D}$  and  $\eta_{sel,m}$  signify the energy price selling to the main grid and DNO respectively.

The optimal operation of a smart residential microgrid based on stochastic MPC has been conducted in the work of [135]. The residential microgrid comprised of renewable energy resources, distributed energy generators, energy storage, electrical vehicle, and smart loads. The uncertainties are modelled in a short – term forecasts of renewable energy generations, load demand, and electricity prices. The proposed method aimed to reduce the total daily operational costs of the microgrid. The simulation results by the authors have shown the economic advantages of the method in comparison to the traditional day – ahead programming approach.

### 5.2.3.3. Multi - scenario MPC implementations in renewable energy.

Multi – scenario MPC utilizes multiple scenario generations within a given optimization horizon to implement a control action at present time,  $t$ . Similar to uncertainty modelling (Refer to Section 3), the independent multiple scenarios generated are synthesized from random input variables of PDFs to produce PDFs of output variables in representing the uncertainties. Ranges of solutions exist, each with its own probability as represented in the output PDFs. The most cost – effective scenario in terms of objective functions are chosen to be the control action within the optimization horizon.

An adaptively constrained stochastic MPC has been proposed in Ref. [136] for optimal dispatch of microgrid. The objective function is formulated to minimize the total operation cost including cost of running generator and cost of purchasing electricity form DG, as expressed in the following equation.

$$\min \sum_{i=1}^T \{c_{con} P_{con}(t+i|t)\} + c_{Grid}(t+i|t) P_{Grid}(t+i|t) \quad (36)$$

Where  $T$  represents the length of time horizon,  $i$  denotes the time step index,  $P_{con}(t+i|t)$  stands for power discharge from the controllable generator in  $i$ -step ahead,  $c_{Grid}(t+i|t)$  denotes the

electricity price for energy exchange in  $i$ -step ahead,  $P_{Grid}(t+i|t)$  is power exchange between MG and DG in  $i$ -step ahead.

The method adaptively/dynamically changed the rate of constraint violation in the microgrid operation to improve the performance of the energy dispatch. In comparison to the robust MPC method, the authors have shown that the method can improve the dispatch performance (less conservative) in cases of uncertain renewable generations and loads. Furthermore, with increment of prediction horizon, computational efficacies were not significantly affected.

Stochastic MPC control strategies in a case of hydrogen – based microgrid have been compared in the work of [137]. The three categories of stochastic MPC mentioned in the previous paragraphs were compared in the work of the authors in an experimental setup including a PEM electrolyser, lead – acid batteries, and a PEM fuel – cell as the main equipment. For each category of the stochastic MPC effectiveness, performances, advantages, and disadvantages were discussed. Authors have discussed extensively the valid criteria needed when selecting the appropriate stochastic MPC method.

#### 5.2.4. MPC's comparison and future trending

**5.2.4.1. MPC's comparison.** It is apparent from the trending of recent stochastic MPC in renewable energy applications that the tree – based and chance – constrained MPC were the most used methods in recent studies. The multi – scenario MPC yielded a robust but over – conservative solutions. Therefore, this category of MPC is not preferred due to the need of an accurate representation of the system, in which the tree – based and chance – constrained MPC could provide better.

Furthermore, in cases of stochastic MPC applications, the prediction/control window is typically within 24 h. Despite the heavy computational expenses of the tree – based compared to multi – scenario MPC, the calculation time within the mentioned window is still relatively inexpensive. The multi – scenario MPC is more suitable in cases of huge numbers of scenarios needed to be considered (i.e. Optimization within 8760 h in a year, 1 – hour time step, and multivariate properties). The multi – scenario MPC could provide a certain robustness of system's representation to the potential disturbances and provide a trade – off between the best solution and the computational expenses.

The chance – constrained MPC offers the lowest computational expenses compared to the other two. It formulates the optimization problem by considering the probabilities of the uncertainties without adding the variables' size. In the work of [137], the chance – constrained MPC outperforms the other MPC methods by offering a reduced computational time, lower operational costs, and minimal energy exchanges with the networks. These advantages of chance – constrained MPC are one of the reasons of frequent usage of this MPC method as shown in recent literatures stated in Table 11. However, the chance – constrained MPC requires an explicit statistical characterization of the systems' disturbances. For the selection of the suitable MPC method, priority factors such as operational costs and computational expenses must be taken into considerations [137]. provided a general guideline in choosing the best stochastic MPC for a given priority factors. Nonetheless, in general categories of stochastic dynamic programming, an efficient method lies often on the specific problems at hand as stated by Ref. [6].

The prediction/control window played an important role in determining the accuracy of the solution as well as computational time. A long prediction/control window would mean a more accurate representation of unfolding events, thus yielding a greater accuracy in finding the best solution. However, the computational expenses increase as the window increases. Trade – offs between prediction/control window and computational expenses must be

determined in order to produce the needed solution.

**5.2.4.2. Future MPC's trending.** The future trends in stochastic MPC are converging towards a multi – scale and multi – time based optimizations as stated in the works of [125,126,129]. In a renewable energy system, where multiple sources of energy generations are present, a realistic representation must consider these sources in order to provide an insight closer to real – world applications. Managing surges of dynamic demands and supplies from plug in EV (V2G), varying behaviours of energy consumers, smart appliances, demand response, and intermittent multiple renewable energy resources are the challenges that must be addressed together in future smart grid – systems. In addition, these challenges are all time – dependant variables in which, each of them possesses traits with dominance in certain time – steps. Addressing the challenges in a multi – time scale approach could capture the undisclosed dynamic behaviour of the system.

In such systems where the dynamics are complex, multivariate, and time dependent, exact solutions are difficult to obtain. Therefore, approximate solutions to such cases are more feasible in the forms of ASDP. The works of [19,138] combines the stochastic method with a (meta)heuristic methods. The stochastic method is hybridized with genetic algorithm to produce ranges of relaxed solutions. Trending in hybridization of stochastic and meta-heuristic methods are relatively new but promising in the field of stochastic optimizations to improve the ASDP algorithms. Interested readers are encouraged to read the works of [2,139–141] for recent reviews of (meta)heuristic methods and intelligent searches in the field of renewable energy applications.

## 6. Conclusions

Stochastic optimizations in renewable energy applications have shown its successful implementations in recent surveys that are presented in the paper. Almost always, based on the works of many authors, the stochastic optimization techniques exhibit enhanced performances and can deliver accurate representations in capturing the uncertainties of renewable systems. Despite its advantages, due to numerous amounts of samplings and unfolding events, which are discussed in the works of many authors to improve or develop novel algorithms in increasing the efficiency of stochastic optimization techniques. Within these contexts, the relevant research themes going into the future based on stochastic optimization algorithms are concluded as follows:

- i. Novel scenario generations and uncertainty modelling approaches; These are necessary in renewable systems integrations where trending in the future involves stochastic multi – scale modelling. With rapid increment of data and size of renewables' problem, perhaps model – driven approaches alone could not fully address and cope with the underlying complexity in vast multivariate and expanding renewable systems. Data – driven scenario generations could provide a pivotal role as highlighted in the works of [41,142].
- ii. Unfolding dynamic uncertainties in multi – stage problems; Addressing dynamic probability issues as scenarios/new forecasts unfolds have been addressed by several authors [103,104] in the paper. Better weather and power forecasts which provide information with dynamic uncertainties as events unfold would incorporate a more robust real – time decision – making strategy for generation companies in handling stochastic renewable generations.
- iii. Implementing new recent notable algorithms in the field of renewable energy optimizations; Recent work by Ref. [143]

- in the form of proximal policy optimization (PPO) has shown great promise in updating multiple epochs per data sample. The method boasts the ability of simple implementations and great stability as well as better overall performance in comparison to its predecessor, trust region policy optimization (TRPO). The PPO algorithm has attracted many authors especially in the field of computational sciences. No works of PPO have been published in renewable energy applications.
- iv. Improvements of existing sampling and decomposition methods; Parameters such as number of scenarios needed, scenario reduction techniques, quality of scenarios generated, and relevant scenarios generated are still under extensive study as highlighted by the literatures in these sections. Acceleration techniques and efficient cuts have been developed by several authors in the decomposition methods approaches to speed up calculations (see [Table 7](#)). Where some sampling and decomposition methods proved to be advantageous, further testing of the methods to other renewable applications are still required.
  - v. Hybridizing existing methods with intelligent search (meta – heuristic method); Especially in problems with higher dimensions (Non – Linear), accurate representation of renewable systems is difficult. Intelligent searches find a relaxed approximation to a solution and can reduce computational expenses while increase accuracy as highlighted in the works of [[19,138,144](#)]. The works of meta heuristic method in renewable energy applications are mainly in the field of deterministic optimization problems [[139](#)].

While algorithms are important in solving the stochastic renewable energy problems, future research areas in this field have also been identified from the surveys conducted. The trending themes moving forward can be broken down in three main categories:

- i. Plug in EVs integration to microgrid; The surge of plug in EVs are expected in the nearest future as these vehicles are more efficient and produces relatively less greenhouse gases [[145](#)]. These EVs will lead to unique future challenges as well as opportunities in future MG systems. The plug in EVs are mostly stochastic problems as charging demand profiles vary from one user to another. Main themes regarding the integration of Plug in EVs in microgrids are identified namely; Charging and scheduling of Plug in EVs [[146–148](#)], renewable energy integration via vehicle to grid operation [[149](#)], and willingness of participation towards the usage of Plug in EVs [[150](#)]. Interested readers are encouraged to read the recent notable works mentioned pertaining to these highlighted themes and its solution methodologies.
- ii. Demand side management (DSM); Multiple authors have started to consider load demand as an active entity as surveyed. Results have shown reduction in peak demands, lower costs, and reduction of generation capacities. However, the success of DSM highly depends on the policies involved and active participation of consumers. Recent review by Ref. [[151](#)] identified the consumers as one of the three main aspects of smart grid management. Authors have also highlighted that the acceptance of DSM varies from one consumer to another. It is critical that the focus of future researches is consumer – centred to improve acceptance in DSM for a better management of the electrical grid. The main future research directions identified regarding DSM and its related recent works are; Consumer engagement methods

[[152](#)], accurate modelling of consumer's behaviour [[142](#)], security and privacy and scalability [[155](#)].

- iii. Multi – scale and multi – time – scale distributed renewable energy systems; vast amount of literatures have supported the claim that having hybrid or combination of renewable systems would allow for a higher fraction of renewable generation in a distributed renewable energy system. However, increasing the scale of distributed generation from housing, to district, and finally to national scale would mean addressing new challenges such as ensuring the grid and market stability in a growing complex socio – techno – economic system with underlying dynamic uncertainties and probabilities. Furthermore, each renewable component, consumer's appliances, and electricity market all have different time – scales in which they are dominant. Addressing both multi – scale and multi – time – scale problems with high penetration of intermittent renewable resources in distributed generation are the future research areas in the field.

The paper highlighted the recent and notable stochastic optimization approaches in renewable energy applications. The advantages and challenges of stochastic optimization methods are carefully evaluated, and its recent trending and future works are summarized in this paper. An intuitive approach was presented to enlighten new researchers in venturing into the stochastic optimization methods within the domain of renewable energy applications.

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